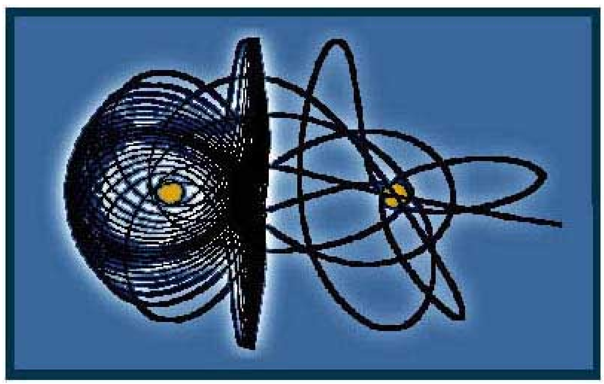
CELMEC IX

an international meeting on Celestial Mechanics



List of POSTERS Titles and Abstracts

Author: Kyriaki I. Antoniadou
Democritus University of Thrace, Department of Physics
Title: *"How dust particles orbiting white dwarfs can reveal undetected co-orbiting exoplanets"*

Abstract. The time evolution of the observed periodicities of transiting rocky debris around white dwarfs (WDs) potentially signposts ongoing dynamical interactions between the debris and co-orbital asteroids or planets with similar orbital periods (1/1 mean-motion resonance). We analyse WD/secondary/dust particle systems within the framework of the circular and elliptic restricted three-body problem (CRTBP and ERTBP). Our secondary masses include Ceres, Mercury, Earth and Neptune so that we can help infer the presence of both large asteroids and planets through our analysis. We study dust particles on both planar and non-coplanar orbits (2D and 3D RTBPs). We firstly compute families of 1/1 resonant symmetric and asymmetric periodic orbits in the 2D-CRTBP, 3D-CRTBP, 2D-ERTBP and 3D-ERTBP. Doing so generates novel results in their own right and diagnostic tools for precise guidance for our next steps.These include two newly-discovered bifurcation points, two branches of asymmetric periodic orbits with a generating orbit at high dust eccentricity value in the 2D-CRTBP and two families of symmetric ones in the 3D-CRTBP. We also discover new families of asymmetric periodic orbits in the high-eccentricity 2D-ERTBP regime.Based on them, we generate detailed phase portraits. Finally, we run suitably seeded N-body simulations to compare with the periodic orbits, and to provide observational links with debris close to WDs. The N-body simulations reveal that over 10 yr, mass analogues of Mercury, Earth and Neptune generate orbital period deviations in the dust of, respectively, $\sim 1-10$ s, $\sim 10-100$ s and over 100 s, when the secondary moves on a circular orbit. When this restriction is lifted off, Mercury-mass planets can regularly generate deviations exceeding $\sim$ 100 s, which potentially aids observational inferences of hidden asteroids or planets orbiting WDs.

This is a joint work with Dimitri Veras (Univ. of Warwick).

Author: Balnur Assan
`Al-Farabi Kazakh National Univ.,`
`Fac. of Mech. and Math., Dept. of Mechanics`
Title: *"Evolutionary Equations of TranslationalRotational Motion of an Axisymmetric Body in the Restricted Three-Body Problem with Variable Mass"*

Abstract. In this work, the translational-rotational motion of an axisymmetric small body with variable mass and variable oblateness in the Newtonian gravitational field of two primary spherical bodies with variable masses is investigated within the framework of the restricted three-body problem in a barycentric coordinate system [1]. The masses of the bodies vary isotropically over time, and no reactive forces arise. The restriction of the problem is characterized by the assumption that the small body does not influence the motion of the two primary spherical bodies.

The differential equations of the secular perturbations of the translational–rotational motion of the system of two primary spherical bodies in the barycentric coordinate system, written in analogues of the Delaunay–Andoyer variables, are analytically exactly integrable for arbitrary laws of mass variation [2].

Thus, the problem is reduced to the investigation of a non-autonomous system of differential equations describing the translational–rotational motion of an axisymmetric small body with variable mass and variable oblateness in analogues of the Delaunay–Andoyer variables.

Expansions of the perturbing function have been obtained, in principle, with any required accuracy, in terms of small parameters corresponding to analogues of eccentricity and inclination. In practice, the expansions of the perturbing function have been computed up to and including second order in the small parameters. In the absence of resonance, by averaging over the fast variables, evolutionary equations of the translationalrotational motion of the small axisymmetric body in the restricted three-body problem with variable masses have been obtained.

The evolutionary equations of the translational–rotational motion of the small axisymmetric body have a complicated form, therefore, it is planned to further investigate them using numerical methods. All analytical computations in this work were performed using the `Wolfram Mathematica`.

This is a joint work with M.Zh. Minglibayev.

References

[1] Minglibayev, M. Zh., Assan B. R. The problem of translational-rotational motion of a non-stationary axisymmetric small body in the gravitational field of two spherical bodies with variable mass. *International Journal of Mathematics and Physics*, **15**(2), 119–126 (2024)
doi:10.26577/ijmph.2024v15i2b12

[2] Minglibayev, M. Zh., Assan B. R. Exact solutions of equations of the two-primary-body problem in the restricted three-body problem with variable masses. *Journal of Mathematics, Mechanics and Computer Science*, **128**(4), 134–147 (2025)
doi:10.26577/JMMCS2025128410

Author: Emma Beligni
`Department of Mathematics and Applications,`
`University of Milano-Bicocca`
Title: *"Natural Transport Phenomena Based on Resonant Periodic Orbits"*

Abstract. Mean-motion resonances are a fundamental dynamical phenomenon in celestial mechanics, arising when two orbiting bodies have orbital periods in a rational ratio.

When considered in a three-body approximation, it is known that resonant periodic orbits can be both stable and unstable, and that their invariant hyperbolic manifolds can be used as dynamical bridges to connect different regions of phase space. Indeed, it is possible to design transfer trajectories on the basis of the heteroclinic and homoclinic connections. This mechanism holds for both natural and artificial bodies.

In this work, we compute resonant periodic orbits in the planar approximation of the Circular Restricted Three-Body Problem, considering both apocentric and pericentric configurations, along with their stability. The results are projected onto a suitable Poincar e map to see the role of the dynamical invariant objects in the connection between different regions.

The applications considered are natural transport phenomena in the Sun-Earth and in the Sun-Jupiter system.

This is a joint work with Elisa Maria Alessi.

Author: Saltanat Bizhanova
Al-Farabi Kazakh National Univ.,
Fac. of Mech. and Math., Dept. of Mechanics
Title: *"On Secular Evolution of Translational-Rotational Motion of an Axisymmetric Body in a Central Gravitational Field"*

Abstract. The long-term evolution of translational-rotational motion of an axisymmetric body with variable oblateness in the central gravitational field is considered. Newtons interaction force is characterized by an approximate expression of the force function up to the second harmonic. The body masses vary isotropically at different rates. The axes of the own coordinate system of the nonstationary axisymmetric body are directed along the principle axes of inertia of the body and we assumed that in the course of evolution their relative orientation remains unchanged. Doing necessary symbolic computations, we obtain the equations of motion of the body in terms of the canonical osculating Delaunay-Andoyer elements. Using the methods of canonical perturbation theory, a particular case of the problem is investigated in detail, when the reactive forces and additional torques are equal to zero. Equations of the translational-rotational motion of the body in osculating analogues of Delaunay-Andoyer elements are described [1]. In this case, the unperturbed motion is analogous of Euler-Poinsot motion – the rotational motion of the free non-stationary axisymmetric body around its own center of inertia. In unperturbed motion, when the ellipsoid of inertia passes through a sphere, in particular, the angular velocity of its own rotation is zero and at the next moment changes sign. In this case, the non-stationary axisymmetric body starts to rotate in the opposite direction. Equation of motion consists of twelve nonautonomous first-order equations that are canonical. In the case when there is no resonance, the evolutionary equation is obtained by double averaging according to the Gaussian scheme. The evolutionary equation decomposes into a system of four first-order differential equations with one first integral, the solution of which determines the evolution of the system [2]. On the basis of these equations establish some qualitative analysis of motion [3]. The resulting secular equations of motion are solved numerically with the Wolfram Mathematica.

This is a joint work with M.Zh. Minglibayev and A. Prokopenya.

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[1] Minglibayev, M.Zh.: Dynamics of gravitating bodies with variable masses and sizes [Dinamika gravitiruyushchikh tel s peremennymi massami i razmerami] (in Russian). LAP LAMBERT Academic Publishing, Germany (2012).

[2] Bizhanova, S.B., Minglibayev, M.Zh. and Prokopenya, A.N.: A Study of Secular Perturbations of Translational-Rotational Motion in a Nonstationary Two-Body Problem Using Computer Algebra. Computational Mathematics and Mathematical Physics, **60**, 26–35 (2020) https://doi.org/10.1134/S0965542520010054

[3] Minglibayev, M.Zh., Prokopenya, A.N. and Bizhanova, S.B.: Analysis of Evolution Equations of a Nonstationary Axisymmetric Body in a Nonstationary Central Gravitational Field. 8th International Congress of Serbian Society of Mechanics Kragujevac, Serbia, June 28-30, 638-647 (2021)

Author: Chetan Bora
Department of Mathematics and Computing,
Indian Institute of Technology, Dhanbad
Title: *"DeepIntercept: A Vision-Guided Deep Learning and PN Control Framework for Precision Kinetic Impactor Missions"*

Abstract. We present an autonomous terminal guidance framework for kinetic-impactor missions that integrates asteroid instance segmentation, centroid-based relative navigation, and three-dimensional proportional navigation (PN) control. A lightweight YOLOv8n segmentation network, trained on spacecraft, synthetic, and ground-based imagery, provides asteroid masks and image-plane centroids that define line-of-sight (bearing) measurements for the EKF in the early terminal phase, before transitioning to LiDAR-derived three-dimensional centroid information. The estimator incorporates innovation-based gating, covariance inflation, and fallback updates to maintain stability under degraded geometry and during the 2D-to-3D centroid handover. Relative motion is propagated using a short-duration terminal dynamics model, while guidance employs a vector PN law augmented with bounded terminal corrections, actuator saturation, quantisation, and first-order command lag. The framework is evaluated through Monte Carlo simulations (N=500) for a nominal stand-off distance of 1000 km and a closing velocity of 6.6 km/s. The segmentation model achieves mAP@0.5 of 99.515066007614n a held-out dataset. In closed-loop simulations, the median final lateral miss is 10.24 m and propellant usage is 23.45 kg, and all runs result in successful impact within the tested envelope. EKF diagnostics indicate broadly consistent estimation, with localized transients near the centroid handover and terminal correction phases. These results demonstrate that vision-based state extraction, filter-based navigation, and PN guidance can be integrated into a practical architecture for autonomous precision intercepts in planetary-defence applications.

Author: Ian Brunton
Caltech
Title: *"On the origin of the Laplace resonance"*
Abstract. We examine the origin of the iconic 4:2:1 Laplace relation between Io, Europa, and Ganymede. Starting from minimal assumptions within the context of the core accretion paradigm for giant planet formation, we illustrate the evolutionary pathways available to establishing the Laplace relation in the circumjovian disk. Upon scrutinizing the collective consequences stemming from primordial Jupiter's inflated quadrupole moment, strong magnetic field, and high accretionary luminosity, we have reached an inescapable conclusion: a stable 4:2:1 commensurability did not exist among the Galilean satellites during Jupiter's disk-bearing epoch. Ultimately, the magnetic coupling between the host planet and its surrounding nebula dictated the radial distance at which the three moons initially converged. In the inner disk, however, a 4:2:1 resonant architecture was inherently unstable, and therefore instead of harmonizing, Io and Europa must have been gravitationally pushed into the disk's MRI-active region by their larger exterior perturber. Ganymede's migration stalled in the disk's turbulent deadzone, while the two smaller moons continued to evolve inward until stabilizing amid the low density active region for the remainder of the disk lifetime. The 2:1 resonance between Io and Europa is likely primordial, but the 4:2:1 relation including Ganymede is not. Our proposed scenario remains independent of whether Callisto ever joined the chain or remained dissonant. We submit that the assembly of the Laplace relation required tidal dissipation, occurring in the epoch following the photoevaporation of the circumjovian and protosolar nebula.

Author: Nelson Callegari Jr.
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Institute of Geosciences and Exact Sciences
Title: *"Secondary Spin-Orbit Resonances: Application to the Irregular Inner Satellites of the Giant Planets"*

Abstract. Over the last four decades, 37 satellites embedded in or located near the ring systems of the giant planets of the Solar System have been discovered by spacecraft and space telescopes. Of these, 32 are irregularly shaped bodies, whose non-spherical figures significantly enhance the gravitational torque exerted by their parent planets on their rotational motion.

In this work, we investigate the rotational dynamics of all these inner, irregularly shaped satellites, focusing on the synchronous rotation regime and searching for secondary resonances in phase space. These resonances arise from commensurabilities between the orbital mean motion and the frequency of physical libration and may occur within the synchronous state for particular values of the satellites' principal dimensions.

Following the methodology of Wisdom (2004), we analyze the rotational dynamics both analytically and numerically. Analytically, we derive a Hamiltonian model for the 2/1 secondary resonance, which is found to occur in 13 satellites. By comparison, the 1/1 secondary resonance is present in 10 satellites, and for this case we adopt the Hamiltonian developed by Callegari (2024). In all cases, the analytical predictions are validated through numerically computed surfaces of section (see also Gkolias et al. 2016). The 2/1 secondary resonance is found for Adrastea and Thebe (Jupiter); Epimetheus, Atlas, Janus, Polydeuces, Helene, Daphnis, and Pan (Saturn); Portia and Cressida (Uranus); and Galatea and Despina (Neptune). The 1/1 secondary resonance occurs for Amalthea and Metis (Jupiter); Methone, Aegaeon, Prometheus, and Calypso (Saturn); Desdemona, Belinda, and Juliet (Uranus); and Naiad (Neptune).

The Hamiltonian models allow us to estimate the libration amplitudes associated with the secondary resonances and compare them with those of the classical synchronous state. We show that the libration amplitude in a secondary resonance is always larger than that of the classical synchronous mode. Although the present rotational states of these satellites remain unknown, our results indicate that, if any of them are trapped in a secondary resonance, they would exhibit large libration amplitudes even under the assumption of a homogeneous internal structure.

Callegari, N. Jr. A Hamiltonian for 1/1 rotational secondary resonances, and application to small satellites of Saturn and Jupiter. Commun. Nonlinear Sci. Numer. Simul., 138, 108–224 (2024).

Gkolias I, Celletti A, Efthymiopoulos C, Pucacco G. The theory of secondary resonances in the spinorbit problem. Mon Not R Astron Soc 2016; 459:1327.

Wisdom J. Spin-orbit secondary resonance dynamics of Enceladus. Astron J 2004; 128:484–91.

Authors: Gregorio Ricerchi and Aurélien Crida
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Nice, UMR7293 Lagrange

Title: *"On the gravitational focusing on (Saturn's) rings"*

Abstract. Saturn's rings are constantly bombarded by interplanetary dust particles, and this phenomenon has a strong impact on their composition (99average zero angular momentum). A key parameter to estimate the magnitude of these effects is the gravitational focusing, that is the ratio of the effective cross section of the rings accounting for the bending of their trajectories by Saturn's gravity to their geometrical cross section.

Here, we show that the expression classically used in the literature is inappropriate to estimate the bombardment rate on Saturn's rings for three reasons:

(i) this formula applies globally to the whole sphere, not locally at a radius r;

(ii) this formula applies to a sphere, not a planar ring;

(iii) the incoming velocity is underestimated.

All in all, we conclude that the bombardment rate suffered by the rings is about 5 times lower than previously thought. This lowers 5 times the effects of the bombardment, and undercuts the arguments in favor of the rings having formed only a hundred million years ago.

Author: Allan Kardec De Almeida Junior

CFisUC, Department of Physics, University of Coimbra

Title: *"Segmentation of the orbit based on overdetermined and continuity constraints"*

Abstract. We present a novel technique to segment the orbit, which can be useful for the orbit determination procedure, but also to a broad range of systems subject to boundary conditions from areas such as biology, physics, chemistry, engineering, etc. The segmentation is followed by the overdetermined and continuity pair of constraints, based on a vectorial formulation of the Theory of Functional Connections. While the continuity constraint ensures analytical smooth connections between the segments, the overdetermined constraints allow analytically embedding constraints with nonlinear components. We apply the technique on transfers between Earth and Moon. The efficiency is demonstrated through several orders of magnitude gains both in accuracy and computational speed in comparison with unsegmented orbits.

The study can be seen in (de Almeida Jr, A.K., Acta Astronautica, 242, 2026).

Author: Guilherme De Oliveira Paes
Federal University of Sao Paulo (UNIFESP)
Institute of Science and Technology (ICT)
Title: *"Quasi-frozen orbits around Mercury: linear stability and bifurcation analysis"*
Abstract. Quasi-frozen orbits, trajectories that remain in the vicinity of equilibria of the orbit-averaged dynamics, undergoing bounded librations about them, are attractive candidates for long-duration science orbits around planetary bodies. We study quasi-polar quasi-frozen orbits around Mercury under a high-degree gravity field (zonal harmonics through degree 8, and sectoral and tesseral harmonics through degree 4), third-body attraction from the Sun, and solar radiation pressure (SRP). Because Mercury's heliocentric orbit is appreciably eccentric, we retain its eccentricity and focus on the two limiting configurations corresponding to perihelion and aphelion. Using canonical perturbation theory, we build a reduced Hamiltonian in Delaunay variables through a Lie-series (Hori-Deprit) transformation, casting the problem as a two-degree-of-freedom system in the canonical pairs (g,G) and (h,H). Its equilibria are located by Newton iteration and classified by linear stability and topological type across the semimajor-axis/area-to-mass-ratio parameter plane. As the parameters vary, the family of equilibria undergoes a Hamiltonian-Hopf bifurcation in which, driven by SRP, a linearly stable centre becomes a complex saddle (focus-focus) as two eigenfrequency pairs coalesce and form a complex quartet. We characterize this transition rigorously via its Krein signature and the associated 1:-1 resonance-condition, confirmed in extended-precision arithmetic. To connect the reduced model to the full dynamics, we invert the Lie transformation to recover osculating initial conditions and propagate them under the full, non-averaged Cowell model over ten years. Periapsis sections of the full dynamics reveal nested invariant curves encircling a common centre, providing the numerical signature of quasi-periodic librations on invariant tori associated with the reconstructed counterpart of the reduced-system Hamiltonian-Hopf transition. This centre is slightly displaced from the reduced-system equilibrium, providing a direct measure of the averaging error. These full-force integrations confirm the quasi-frozen character of the reconstructed orbits, since the osculating eccentricity and inclination remain bounded and the periapsis stays well above the surface over the whole span.

Author: Barnabas Deme
Baja Astronomical Observatory of SZTE University
Title: *"Misaligned rings around minor planets with moons"*
Abstract. Recent observations have confirmed the existence of rings around minor bodies in the outer Solar System. These objects may possess satellites as well. I analytically investigate the interaction between such rings and satellites. I show that the perturbations from the moons may efficiently lead to off-equatorial rings around minor bodies like trans-neptunian objects or centaurs. In particular, I derive criteria for the orbital elements under which such misaligned rings may exist.
Reference: Universe 2026, 12(4), 108.

Author: Pierfrancesco Di Cintio
CNR-ISC, INAF-OAA + INFN-Firenze

Title: *"Information entropy and Lyapunov exponents in orbital dynamics, revisited"*

Abstract. The problem of detecting and quantifying chaos in gravitational systems is related to the computation of Lyapunov exponents and dynamical entropies. The complex structure of phase-space in many degrees of freedom systems, often prevent to correctly evaluate these chaoticity indicators. Partially due to their slow numerical convergence and partially for problems related to numerical accuracy. In this work we adopt a definition of information entropy, related to the dynamical KS entropy that can be evaluated directly from the structure of the simulated data as stored in a binary file, rather than computed on the actual phase-space coordinates therein encoded. Exploiting the relation to the dynamical entropy we therefore obtain a prediction of the scaling of the (maximal) Lyapunov exponents with the systems parameters such as the orbital energy. As a test, we use to simple and well studied low-dimensional models, namely the Henon-Heiles system and the circular restricted three-body problem, both showing a mixed phase-space with a tangled structure of nearly regular and chaotic orbits. We evaluate the Lyapunov exponents and the data-based information entropy as a function of their conserved quantities. Moreover apply the same procedure to single particle orbits in time-dependent $N-$body potentials with increasing degrees of freedom at fixed scaled energy.We find that the information entropy has a scaling with the system's energy compatible with that of its largest Lyapunov exponent. A similar trend is also observed for single particle orbits in many body models. Moreover, the data-based entropy converges more rapidly for fixed integration time than the Lyapunov exponents for the case of sticky orbits, typically associated to large values of λ converging over longer time intervals. We speculate that H can be used as a valid chaos indicator in absence of a well defined tangent space dynamics or in case of slowly convergent Lyapunov exponent time series. Moreover, we discuss the possibility to apply such method to the determination of the chaotic-instability time scales for the orbits of minor bodies of the Solar System or artificial satellites of which the phase-space is naturally well determined due to the large number of orbit points available.

Author: Fredy Leonardo Dubeibe
Universidad de los Llanos
Title: *"On the Use and Limitations of Perturbed Restricted Few-Body Models in Celestial Mechanics"*

Abstract. In recent years, a growing number of studies have focused on modified versions of the restricted three- and four-body problems, incorporating additional perturbations such as non-standard Coriolis terms, relativistic corrections, drag forces, or ad hoc potential modifications. While these extensions are often motivated by physical considerations, their direct numerical implementation frequently leads to the loss of dynamical stability of the classical configurations, particularly when the full equations of motion are allowed to evolve self-consistently. Despite this, such models remain widely used because they are commonly embedded within predefined restricted frameworks in which the positions of the primary bodies are artificially fixed. This assumption suppresses the back-reaction of the perturbations on the primaries, thereby preserving the nominal geometry of the system and enabling the continuation of the analysis. However, this practice raises important conceptual and physical concerns regarding the consistency, validity, and interpretability of the resulting dynamics. This poster critically examines the mathematical and physical implications of introducing perturbations into restricted few-body models with frozen primaries. We discuss how this approach can mask intrinsic instabilities, potentially leading to misleading conclusions about orbital structure, equilibrium points, and long-term behavior. The analysis highlights the need for greater care in distinguishing between mathematically convenient restricted formulations and physically meaningful dynamical systems, particularly when extending classical models beyond their original assumptions.

Author: Valentin Etienne
Observatoire de Paris - LTE
Title: *"Libration and Cassini States of Celestial Bodies with a Viscous Spherical Core: A New Analytical Framework"*
Abstract. Analytical models describing the rotation of celestial bodies with a viscous fluid core commonly assume a simple motion for the fluid – a solution originally derived by Hough and Poincar for a perfect (inviscid) fluid. In these models, viscosity is subsequently introduced through a friction torque at the core–mantle interface.

We propose a new analytical model in which the fluid core motion is described more realistically through a solution of the linearized Navier-Stokes equations. Our methodology is inspired by the work of Ievleva, who studied the motion of a pendulum containing a spherical cavity filled with an incompressible viscous fluid. We will examine the effects (similarities and differences) of this change on planetary rotational dynamics.

Author: Maria Julia Fassis
UNESP, IGCE, Department of Physics
Title: *"A study of the retrograde families of symmetric periodic orbits at the 1/-1 and 1/-2 resonances in the CR3BP varying the mass ratio"*

Abstract. We computed retrograde symmetric periodic orbits in the planar CR3BP at the 1/-1 and 1/-2 resonances for values of the mass ratio in the set

$$\{0.001; 0.0121506683; 0.04; 0.1056338028; 0.3; 0.4; 0.5\}\ .$$

We observed that at small mass ratio there are three modes of 1/-1 resonant families (named mode 1, and outer/inner mode 23) in agreement with previous studies [Morais and Namouni (2019), Voyatzis et al. (2025)]. The mode 23 families bifurcate from the circular families (inner and outer branches) when the horizontal stability index K2D=-2. We showed that this bifurcation occurs on the inner circular family for all values of mass ratio, but from the outer circular family it happens only for mass ratio ≤ 0.32. Our results also show that the width of the gap that divides the retrograde circular family at the 1/-1 resonance location in outer and inner branches increases with mass ratio. Moreover, we identified bifurcations with period multiplication 2 to 6 from the mode 1 resonant 1/-1 family and from the mode B resonant 1/-2 family in the systems with mass ratios 0.04, 0.1056338028 (Pluto-Charon) and 0.3. This work has been submitted for publication [Fassis and Morais (2026)].

Author: Szilveszter Fehér
Eotvos University, Institute of Physics
Title: *" Floquet-Theory-based Stability Analysis of the Elliptic Relative Equilibria Solutions of the Gravitational n-Body Problem"*

Abstract. Elliptic relative equilibria solutions are explicit, exact analytical solutions of the gravitational n-body problem, where the bodies travel on elliptic Keplerian orbits while maintaining a self-similar configuration known as a central configuration. A general Floquet-theory-based linear stability analysis formalism of the elliptic relative equilibria solutions is presented and applied to find the linear stability diagrams belonging to the studied central configurations (Lagrangian equilateral triangle, 1+N-gon, and the convex four-body kite configuration). Some other configurations (N-gon, concave four-body kite, rhomboid five-body configuration) were found to be totally unstable for every eccentricity. The linear stability results are compared to the nonlinear simulation of the original n-body equations to see whether the stability criteria for the original nonlinear system are the same as what we get from the Floquet stability analysis.

Author: Sylvio Ferraz Mello
Universidade de Sao Paulo,
Instituto de Astronomia, Geofisica e Ciencias Atmosfericas
Title: *"Rotational synchronization and desynchronization of terrestrial planets in the habitable zone of solar-type stars. The case of Venus"*

Abstract. In Venus, the gravitational tidal braking torques are offset by acceleration torques due to the extreme thermal deformation of its atmosphere, capable of keeping the planet spinning backward. But how can a planets rotation become retrograde? The study of Venus' rotation with a new version of the creep tide theory revealed that, in the absence of the atmospheric accelerating torque, Venus' current rotation would become direct again in a few million years. Therefore, its retrograde rotation cannot be attributed to a random event (as a collision). The mathematical analysis of the equations describing the effects of the two torques shows that the phase space of these equations exhibits a feature common to many first-order differential equations: a pitchfork bifurcation. Without an atmosphere, the equation presents a synchronous attractor: tides tend to synchronize rotation and orbital motion. With a dense atmosphere, this attractor bifurcates into two asynchronous attractors: a subsynchronous and a supersynchronous one. The rotation of the primitive planet with only a thin atmosphere, subject to strong tides, will tend to become synchronous. If a dense atmosphere subsequently forms through outgassing, the accelerating atmospheric torque increases, and the planet's rotation reaches the bifurcation. Then, with almost equal probabilities, it evolves towards one of the two asynchronous attractors, one of which may subsequently become retrograde. The simplicity of this evolutionary process suggests that many Earth-like exoplanets in or near the habitable zone of solar-type stars may exhibit retrograde rotation.

Reference: Astronomical Journal (2026)

Author: Aiken Kosherbayeva

Al-Farabi Kazakh National University

Title: *"Differential Equations of Motion in the Three-Body Problem with Variable Masses in the Presence of Reactive Forces in Barycentric Reference Frame"*

Abstract. In this work, the three-body problem (point mass) with non-isotropically varying masses is investigated in the presence of reactive forces within various coordinate systems.

For the first time, the differential equations of motion for the three-body problem with non-isotropically varying masses in the presence of reactive forces have been derived in the barycentric coordinate system.

Furthermore, the invariants of the center of mass have been established in the barycentric coordinate system. These invariants are the single analytical relations that connect the coordinates and velocities of the bodies under arbitrary laws of mass variation.

The resulting differential equations of motion in the barycentric coordinate system and the invariants of the mass center can be used in various qualitative, analytical, and numerical studies of the three-body problem with variable masses in the presence of reactive forces.

The differential equations of motion in the three-body problem with non-isotropically varying masses in the presence of reactive forces in barycentric coordinate system will be investigated using perturbation theory methods based on aperiodic motion over a quasi-canonical section [1], [2].

Herewith, the invariants of the mass center will be used as control analytical relations.

The resulting differential equations of motion in the barycentric coordinate system and the invariants of the mass center can be used in various qualitative, analytical, and numerical studies of the three-body problem with variable masses in the presence of reactive forces.

The differential equations of motion in the three-body problem with non-isotropically varying masses in the presence of reactive forces in barycentric coordinate system will be investigated using perturbation theory methods based on aperiodic motion over a quasi-canonical section [1], [2].

Herewith, the invariants of the mass center will be used as control analytical relations.

This is a joint work with M.Zh. Minglibayev and A. Prokopenya.

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[2] Minglibayev, M. Zh., Prokopenya, A.N. and Kosherbayeva, A.B.: Secular evolution of circumbinary 2–planet systems with isotropically varying masses. MNRAS, **530**, 2156–2165 (2024) doi:10.1093/mnras/stae1015

Author: Matheus Lazarotto
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`da Universidade de Sao Paulo (IME-USP)`
Title: *"An angular-momentum preserving dissipative model for the point-mass N-body problem"*
Abstract. A simple model emulating energy dissipation due to tidal effects is proposed. In it, forces acting between masses remove energy but preserve the total angular momentum of the system. We study the effect of such forces on the particular family of orbits in central configurations, and show that a specific dependence on the mutual distances between the bodies leads to homographic equations equivalent to those of the two-body problem with dissipation. We then describe in detail the topology of solutions of the dissipative two-body system via Poincaré compactification.

Author: Vitor Martins de Oliveira
Institute of Space Research, DLR
Title: *"RheoVolution: a software for the tidal evolution of celestial bodies with complex rheological models"*

Abstract. Tidal dissipation is one of the main mechanisms for energy loss in planetary systems. In this work, we introduce the software RheoVolution, which simulates the motion of N massive triaxial and deformable interacting bodies with a rheology based on the generalized Voigt viscoelastic model. We start by presenting the main ideas behind modeling planetary tides via a time-domain description of the body's deformation and we later show the dynamical equations derived in this theory. We finish with a couple of examples of dynamical phenomena in the Solar System where tidal effects are important.

Author: Ana Julia Mendes Pereira dos Santos
Sao Paulo State University (UNESP),
Institute of Geosciences and Exact Sciences (IGCE)
Title: *“Mapping of Orbital Resonances in the Inner Satellites Systems of Jupiter and Neptune”*

Abstract. The study of orbital dynamics of natural satellites provides fundamental information about the evolution of planetary systems and the mechanisms involved in their gravitational interactions. The small inner satellites of Jupiter and Neptune exhibit behaviors sensitive to the planet’s gravity, its oblateness, and the perturbations exerted by neighboring moons.

The present work aims to investigate possible orbital resonances through a one-dimensional mapping of the semi-major axis space, constructed from numerical simulations by varying the semimajor axis around the position of each satellite and analyzing long-term integrations in the frequency domain. The study employs numerical methods and techniques developed by Callegari & Yokoyama (2010), adapted to the gravitational particularities of the Jovian and Neptunian systems. Each mapping allows us to locate the real position of the satellite within the dynamical structure, identifying whether it is associated with a specific resonance or close to some resonant region. For the Jovian system, the proposal by Hamilton et al. (2001) suggests that the inclinations of Amalthea and Thebe are the result of ”kicks” caused by mean-motion resonances with Io, as this satellite migrated outward due to tidal forces. For the Neptunian system, Brozovi et al. (2020) describe the 73:69 inclination resonance between Naiad and Thalassa. We seek to construct dynamical maps for this resonance, in addition to stability spectra and the characterization of resonant angles.

Author: Andrea Moretti
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Title: *"Tilting Neptune during planetary migration"*
Abstract. Neptune's current obliquity of 28.3° poses a long-standing puzzle: giant planets are expected to form with near-zero axial tilts, so some post-formation mechanism must be responsible for their current tilts. Capture into spin-orbit resonances has been shown to be a viable mechanism to explain the obliquities of Jupiter, Saturn, and Uranus. We investigate whether the same process can account for Neptune's tilt, proposing that it was tilted during planetary migration through capture into a spin-orbit resonance enabled by a now-lost primordial satellite system.

We model the secular dynamics of Neptune's spin axis, accounting for the contribution of a primordial moon system to the planet's precession constant. Through numerical simulations of planetary migration, we track the evolution of Neptune's obliquity and explore the range of satellite configurations that lead to successful resonance capture. We identify an extended region of parameter space – in terms of satellite masses and semi-major axes – that produces the correct obliquity. A representative configuration consists of an inner moon of mass $5\times 10^{-5}\ M_N$ at 35 R_N and an outer moon of mass $3.5\times 10^{-4}\ M_N$ at $100R_N$. These conditions are consistent with recent tilting scenarios proposed for Uranus, suggesting a possible common origin for the obliquities of the two ice giants.

We also explore the possibility of close encounters with planetesimals from the protoplanetary disk during the last stages of migration as an exit mechanism from the spin-orbit resonance, and the connection with the capture of Triton.

Author: Gabriela Rufino Travassos
Aeronautics Institute of Technology (ITA)
Flight Mechanics Department
Title: *"A Planar Circular Restricted Three Body Model for the Small Bodies Population for HD 159243 system "*

Abstract. Small bodies are essential for understanding the present-day dynamics and the evolutionary pathways that led to the current configurations of our planetary system. While dynamical structures arising from the circular restricted three-body problem (CR3BP) govern key transport mechanisms, such as escape, capture, and the fate of trajectories in subsystems like the Trojan asteroids of Jupiter, the Earth-Moon system, and the transport between the Jovian moons, there exist interplanetary regions of the Solar System where these structures are not the primary drivers of the dynamics. In contrast, CR3BP-like mechanisms may play a more dominant role in less crowded planetary systems, particularly those hosting massive planets in close proximity to their host star. One such example is HD 159243, a solar-type star hosting two Jupiter-like planets at orbital distances of less than one astronomical unit. In this system, the considerable mass ratios between the star and each planet, combined with their compact configuration, suggest a substantial contribution from three-body dynamics. Furthermore, the proximity of the planets may allow for overlapping invariant manifold structures with similar energy values, potentially giving rise to heteroclinic connections that act as transport channels between the primaries' regions. During this presentation we will show a Planar Restricted Three-Body Problem (PCR3BP) model for HD 159243 planetary system serving as an example of other compact two-Jupiter lie planets planetary systems. Our model uses the strategy of the patched-three body to connect invariant manifold tubes of Lyapunov orbits of the system creating transfer channels between the planets and testing the long-term conservation of these orbits.

Authors: Muhammad Shoaib, Brahim Benhammouda, Bonnie A. Steves and Winston L. Sweatman

Contact: Bonnie Steves at University of Glasgow,
School of Physics and Astronomy

Title: *"Central configurations in the general four body coplanar problem with different masses"*

Abstract. A generic method for finding central configurations in the general four body coplanar problem with four different masses is derived. New families of non-symmetrical central configurations with non-equal masses are found and all known symmetrical four body non-collinear central configurations are shown to be derivable subsets using this generic model. In this model, three masses are distributed on the vertices of a triangle. Then the fourth mass is allowed to be at any other point in the plane forming either convex or concave central configurations. In this general setting, a necessary condition is derived for the existence of central configurations and for requiring the masses to be positive. Using both analytical and numerical techniques, regions of possible central configurations can be derived. The special cases of four body central configurations investigated include isosceles trapezoids, right trapezoids, right quadrilaterals, convex kites and concave kites. In most of these special cases, the number of parameters of the problem can be reduced from four to two and hence the regions of existence can be shown graphically. A number of cases both convex and concave with no symmetry restrictions or equality of masses were also investigated to demonstrate the generality of the method for finding central configurations in the general four body coplanar problem with four different masses.

Authors: Andras Szell and Bonnie Steves
`Contact: Bonnie Steves at University of Glasgow,`
`School of Physics and Astronomy`
Title: *"Families of periodic orbits in the Caledonian Four Body Problem"*
Abstract. We find families of periodic orbits in the Caledonian Symmetric Four Body Problem (CSFBP), a reflection–symmetric gravitational system with rich dynamical structure. Using a Poincaré return map defined on a reduced phase space, we construct a scalar function K whose vanishing corresponds to periodic motion. Applying a grid-based root-finding algorithm, we identify families of periodic orbits in the equal-mass case ($\mu = 1$) and a non-equal mass case $\mu = 0.1$), classified into two hierarchical types: "12" orbits (binary pairs orbiting each other) and "23" orbits (two single bodies orbiting a central binary). A linear stability analysis based on the eigenvalues of the Jacobian matrix confirms where the periodic orbits are linearly stable. The findings demonstrate strong potential for generalization to higher-order orbits and symmetric $2n$–body Caledonian systems. The periodic solutions also offer dynamical templates relevant to observed quadruple stellar systems.

Author: Pan Tan
Observatoire de la Cote d'Azur
Title: *"Global dynamics of overstable librations in planar three-body problem"*
Abstract. Planets are believed to migrate during their formation due to the planet-disk interaction. For multi-planet systems with convergent migration and substantially damped eccentricities, the high capture probability by first-order mean motion resonance is inconsistent with the paucity of resonant configurations in observations. The instability of the first-order resonances under disk-torque perturbations may be the key to solve the inconsistency. Both the local and global dynamics of overstable librations in a first-order resonance of the restricted and the non-restricted three-body problem are studied in this paper. An analytical treatment based on the classical averaging method in terms of action-angle variable is achieved for the adiabatic case of suiciently weak dissipation: (1) the well-known analytical criterion of overstability is reformulated and the derivation appears simpler than that obtained by linearization of the complete system; (2) an explicit equation for the location of a limit cycle is derived in terms of complete elliptic integrals and solved numerically; (3) the capture probability for separatrix crossing driven by disk-torque is computed explicitly. As the dissipation strength increases, we find that the limit cycle experiences a cascade of period-doubling bifurcations, a well-known process leading to chaos. The critical dissipation strength for the disappearance of limit cycle is estimated first by a perturbative computation of the oscillation of the limit cycle, and then by the limit of the sequence of period-doubling bifurcations. We report the phenomenon of multistability when the dissipation strength is large but smaller than the critical value: the limit cycle and the resonance escape are the two attractors, and the boundary between their basins of attraction is complex but appears deterministic.

Author: Timothée Vaillant
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Title: *"A novel procedure for orbit determination"*

Abstract. The increasing number of resident space objects orbiting the Earth represents a threat for the current and future space infrastructures. To prevent the collision risk, tracking is then necessary to identify them and determine their trajectory and physical characteristics via a process called orbit determination that uses observation data of a space object to estimate its orbit.

Orbit determination methods were initially developed to obtain the orbit of the minor bodies of the solar system from optical observations notably with the works of Gauss for Ceres.

Since then, new observational means have been developed (radar, laser), and numerous different methods have been then developed using each a specific data set.

Indeed, depending on the type of sensors available for observation (radar, optical, laser), different data sets can be obtained (range, angle, range-rate, angle-rate).

However, the constant increase of resident space objects represents an important challenge, and there is then a need for the development of new numerically efficient orbit determination methods.

The Theory of Functional Connections (TFC) is a numerical framework solving constrained differential equations by embedding their constraints in the equations of motion.

The search of solutions is then limited to those exactly satisfying the constraints, which improves the convergence and decreases the computational time.

In this presentation, we will present a new orbit determination method currently under development that uses TFC to embed observation data in the equations of motion guaranteeing then the numerical efficiency of the process.

This new method can process different data sets and is then not limited to a specific one. We will indicate its possible applications, and the future steps to achieve to obtain an orbit determination program similar to the existing software.

Author: Tudor Vartolomei
Alexandru Ioan Cuza University of Iasi, Faculty of Mathematics
Title: *"Artificial Neural Networks for Orbital Stability and Proper Elements of Space Debris"*

Abstract. This work presents a neural-network framework for studying the dynamical stability of Hamiltonian systems that describe orbital motion, with applications to artificial satellites and space debris. The proposed approach combines machine learning methods with classical perturbation theory by learning the relationships between orbital parameters and stability of the system. The obtained model provides a fast tool for identifying stable and unstable regions of phase space, while reducing the computational cost of traditional methods. A complete workflow is developed, from the generation of synthetic training datasets to the analysis of real orbital cases. The datasets are built using parameters of normalization and trajectories of reduced Hamiltonian models, allowing the network to learn complex dynamical features such as resonances and chaotic behavior.

As a complementary study, a second neural-network architecture is proposed for the computation of synthetic proper elements. Proper elements are key quantities used to describe the long-term evolution of orbits and to identify groups of dynamically related objects. The proposed data-driven approach estimates these elements directly from orbital data, offering an efficient alternative to conventional techniques while remaining suitable for large datasets.

Author: Rodolpho Vilhena de Moraes
Federal University of Sao Paulo (UNIFESP),
Institute of Science and Technology (ICT)
Title: *"Frozen orbits around asteroid 99942 Apophis"*

Abstract. The exploration of small bodies in the Solar System requires high-precision dynamical models for the identification of stable orbital regions. This work presents an analysis of frozen orbits around the asteroid 99942 Apophis using canonical perturbation methods. The analytical stage is carried out within the symcelmech framework (Maxima), which performs the Delaunay normalization of the Hamiltonian system through the Hori-Deprit method. A gravitational potential model incorporating the coefficients J2, J3, J4, and the sectorial term C22 is adopted in order to account for the asteroid's non-sphericity and mass asymmetry. To handle the explicit time dependence introduced by the C22 term, an expansion of the phase space is implemented via the homogeneous formalism, thereby converting the system from non-autonomous to autonomous. Following the order reduction of the system, equilibrium points associated with frozen orbits are determined. Subsequently, corrections related to the time dependence are obtained through the transformation equations from mean to osculating variables, yielding initial conditions for frozen orbits. These conditions are then validated through numerical integrations of the non-averaged dynamics.

Author: George Voyatzis
Aristotle University of Thessaloniki, Dept. of Physics
Title: *"On the dynamical environment of a precessing body "*
Abstract. We study the orbital dynamics of small bodies around a precessing oblate spheroid. We analyze the stability of circular orbits under the perturbation parameters that are associated with the amplitude and the frequency of precession.