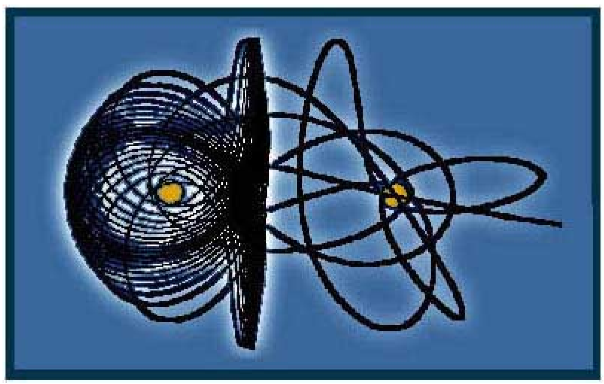
CELMEC IX
an international meeting on Celestial Mechanics



List of TALKS
Titles and Abstracts

Author: Alain Albouy

`Observatoire de Paris, CNRS, LTE`

Title: *"An improved result about the change of time in the equations of the dynamics"*

Abstract. The planar harmonic oscillator may be mapped onto the Kepler problem in such a way that the centered ellipses are sent onto Keplerian ellipses. The body makes two turns on the Keplerian orbit with a non-Keplerian law of time. This map has many surprising features.

At the end of the 19th century, a vaguely stated conjecture emerged from this example and from Appell's projective dynamics (see [1]). Thomas [2] published a proof in 1946. His conclusion is stated in a technical way. I will reformulate and prove the conjecture in the following form. If there is a map from a "natural" mechanical system onto another natural mechanical system where the time is changed from s to t, then $\mathrm{d}s/\mathrm{d}t$ is a function of the position optionally multiplied or divided by the square root of a quadratic first integral.

My "natural" class includes the systems with a quadratic kinetic energy and a potential. It is wider than the one chosen by Thomas, and gives a simpler proof.

[1] Painlevé P., Sur les transformations des équations de la dynamique, Comptes Rendus, 1896, vol. 123, pp. 392-395.

[2] Thomas, T.Y. On the Transformation of the Equations of Dynamics, Journal of math. and phys., 1946, vol. 25, pp. 191-208.

Author: Aya Alnajjarine
LTE / Observatoire de Paris

Title: *"Laplace–Legendre expansion of the planar planetary Hamiltonian"*

Abstract. The disturbing function plays a central role in understanding the long-term dynamical evolution of planetary orbits. Its expansion has traditionally relied on two classical approaches, each adapted to a different dynamical regime. Expansions in eccentricities and inclinations, based on Laplace coefficients, are well suited to nearly circular and compact configurations, while expansions in the semi-major axis ratio, based on Legendre polynomials, are more appropriate for hierarchical systems at arbitrary eccentricities and inclinations. We introduce a hybrid expansion of the disturbing function that combines elements of the classical Laplace and Legendre developments. The formulation retains the structure of the Laplace expansion, but expresses the inverse of the mutual distance as a series whose terms preserve an exact dependence on both the eccentricities and the semi-major axis ratio. We develop this approach for the planar three-body problem and use it to construct the first-order secular Hamiltonian, relevant for modeling the long-term evolution of planetary systems. We assess the convergence of the expansion numerically and compare it with the classical Laplace and Legendre series across a range of orbital configurations. The results show that the proposed expansion provides consistent behavior across different regimes and recovers the expected performance of the classical approaches in their respective domains.

Author: Kyriaki I. Antoniadou
Democritus University of Thrace, Department of Physics
Title: *"Periodic orbits close to exoplanets in resonant chains"*

Abstract. Hitherto more than 1000 multiple planet systems have been discovered by space missions and ground-based telescopes, while many systems possess planets in (or near) resonant chains. Despite the efficient fitting methods, the deduction of all orbital elements, especially the mean anomalies and the longitudes of pericentres remains challenging, while the masses and the eccentricities are often provided with very large uncertainties impacting the long-term regular evolution of each system.

Here, we use the General 4-Body Problem (G4BP), we present different continuation schemes and we compute the families of periodic orbits in resonant chains along with their linear stability for different mass-ratios of the exoplanets locked in resonant chains. As the stable periodic orbits constitute the backbone of stability domains in phase space, we also compute maps of dynamical stability, initialised by the stable periodic orbits in the dynamical neighbourhood of the systems and unravel the boundaries, i.e. the deviations from the nominal observational values, of possible domains where such systems can be hosted for long-time spans. This way we either validate or further constrain the large uncertainties of the observational data or suggest suitable orbital elements. We also provide scenarios of regular evolution found in three-planet systems namely, a combination of two-body mean-motion resonances, or a three-body Laplace like resonance or a combination of a secondary resonance inside a mean-motion resonance within the inner and outer pair of planets.

Author: Miquel Barcelona Poza
`Universitat de Barcelona`

Title: *"On the graph transform method to compute invariant manifolds attached to whiskered invariant tori on Poincaré Maps"*

Abstract. We introduce a semi-analytical method for computing high-order parameterizations of invariant manifolds attached to invariant tori of Poincaré maps. The method is based on an explicit graph transform formulation in which the hyperbolic variables are expressed as functions of the central ones.

A key assumption is the reducibility of the invariant torus, which allows the linearized dynamics along the torus to be conjugated to a system with constant coefficients through a Floquet transformation. In this setting, the normal dynamics of the torus can be effectively decoupled. The computational approach is recursive, using jet transport and solving a set of cohomological equations at each order using Taylor-Fourier series.

This methodology is applied to study natural transport mechanisms on the Earth-Moon system. In particular, we study how it can be modified by means of an oscillating solar sail, allowing for low-energy transfer trajectories.

Author: Alexander Batkhin
Technion - Israel Institute of Technology,
The Stephen B. Klein Faculty of Aerospace Engineering
Title: *"Bifurcations of periodic solutions to Hamiltonian systems with different types of symmetries"*

Abstract. For an autonomous Hamilton system with two degrees of freedom in a generic case, normal form (NF) and power geometry methods are applied to study bifurcations of periodic solutions for various groups of discrete linear automorphisms of the phase space. A Hamiltonian NF is constructed in the neighborhood of a periodic solution, which, with the help of an additional canonical transformation, is reduced to an autonomous Hamiltonian system with one degree of freedom and one small parameter (reduced NF). The equilibrium positions of the reduced NF correspond to families of periodic solutions, and the various bifurcation scenarios essentially depend on the symmetries of the initial system.

Author: Giulio Baù
University of Pisa
Title: *"Long-term evolution of the eccentricity vector for near-circular low lunar orbits"*

Abstract. Low Lunar Orbits (LLOs), characterized by altitudes lower than 100 km, are strongly influenced by the Moon's gravity field. The presence of "mascons" [1] can cause instabilities of the motion and lead satellites to collide with the lunar surface. Therefore, high-degree truncations of the gravity field have been considered essential since the introduction of the first high-resolution models [2]. In this work, we propose a simple solution for predicting the long-term evolution of the mean (i.e. obtained from Kaula's expansion [3] after removing all terms dependent on the mean anomaly of the satellite) eccentricity vector of near-circular LLOs under the effect of the Moon's gravity field. The proposed solution represents an extension of the linear (with respect to the eccentricity) theories of G.E. Cook [4] and R.A. Cook [5] for the long-term evolution of the non-singular projections $\xi = e\cos\omega$, $\eta = e\sin\omega$, where e is the eccentricity and ω the argument of pericenter. [4] considers the J_2 and all odd zonal harmonics and finds that the solution describes a circle in the (ξ, η) plane, whose center is displaced from the origin along the η axis. [5] improves this result by taking into account all zonal harmonics: solutions can show either an exponential or periodic behaviour, and, in the latter case, ellipses are described in the (ξ, η) plane. We include in the framework of Cook's theories the effects of the sectorial and tesseral harmonics. The accuracy of the new solution for LLOs is evaluated through numerical tests. We have recently discovered that a similar result was obtained in [6], where a different method is used for deriving the solution, and the proposed numerical tests deal with near-circular low Earth orbits.

This work is in collaboration with Ana S. Rivero and Rafael Vazquez, both of the University of Seville.

References

[1] P.M. Muller, W.L. Sjogren, Mascons: Lunar Mass Concentrations, Science, 161, 3842, 1968.
[2] A. Konopliv, et al. A High Resolution Lunar Gravity Field and Predicted Orbit Behavior, Advances in the Astronautical Sciences, 85, 1994.
[3] W.M. Kaula, Theory of Satellite Geodesy, Dover, 2000.
[4] G.E. Cook, Perturbations of near-circular orbits by the Earth's gravitational potential, Plantary and Space Science, 14, 1968.
[5] R.A. Cook, Long-term behavior of near-circular orbits in a zonal gravity field, Advances in the Astronautical Sciences, 76, 1992.
[6] F. Deleflie, G. Métris, P. Exterier, An analytical solution of the Lagrange equations valid also for very low eccentricities: influence of a central potential, Celestial Mechanics and Dynamical Astronomy, 94, 2006.

Author: Justine Bodart

University of Namur - Department of Mathematics

Title: *"General AMD–stability criterion for multi-planet systems"*

Abstract. The increasing discovery of exoplanetary systems has made it necessary to study their stability. In this work, we present a generalized framework for evaluating the long-term stability of compact multi-planet systems, based on the AMD-stability criterion defined by Laskar and Petit (2017), which establishes a critical value for the AMD below which close encounters are avoided and the system can be considered stable. The new AMD framework extends the resonance overlap criterion previously proposed by various authors (e.g. Deck et al., 2013; Hadden and Lithwick, 2018; Petit et al., 2020; Tamayo et al., 2021). In this approach, we emphasize the importance of accounting for secular diffusion, which can impact the stability of the system and significantly shift the location of the chaotic boundary. We evaluate the performance of the proposed framework on several compact three-planet systems in first-order resonance, demonstrating good agreement with numerical results. Our general AMD-stability criterion provides a practical tool for quickly and easily assessing the stability of multiple systems.

This is a joint work with Anne-Sophie Libert and Antoine C. Petit.

Author: Chetan Bora
Department of Mathematics and Computing,
Indian Institute of Technology Dhanbad
Title: *"Long-Term Dynamical Evolution and Ejection of Near-Earth Asteroids"*
Abstract. Long-term integrations of asteroid orbits with high-accuracy numerical integrators are essential for understanding dynamical evolution and ejection from the Solar System, but are computationally expensive. Here, we investigate the dynamical behaviour of asteroids and explore machine-learning (ML) and deep-learning (DL) approaches as efficient, scalable alternatives for classifying long-term dynamical outcomes. While the ML classifiers are trained on initial orbital elements, the convolutional neural network is trained on recurrence plots derived from short-period numerical integrations generated with the MERCURY integrator. Ensemble tree models perform strongly on the ephemeris input, and the neural network captures temporal signatures of chaotic motion with comparable or slightly improved accuracy. Backward integrations reveal partial overlap between forward- and reverse-ejected sets, illustrating time-asymmetric behaviour in chaotic regions; these backward results are interpreted only as diagnostic probes rather than reconstructions of past histories. Non-ejected asteroids largely correspond to known dynamical groups, underscoring the constraining role of initial orbital configuration. These methods provide scalable frameworks to complement numerical integrations and inform prioritisation for detailed long-term dynamical studies, with implications for planetary-defence analyses.

Author: Natasha Bosanac

`University of Colorado Boulder, Smead Aerospace Engineering Sciences`

Title: *"Data-Driven Summary of Spacecraft Trajectories in a Multi-Body System"*

Abstract. This talk will present an overview of my research group's approach to generating data-driven summaries of the complex solution space for trajectories within the Earth-Moon system. First, we use differential geometry of curves and surfaces to sample trajectories across the solution space; I will also share recent advances to perform this sampling adaptively. We also use differential geometry to discretize and describe each continuous trajectory using finite dimensional feature vectors that capture its shape. Next, we use a distributed and density-based clustering scheme to construct groups of geometrically similar trajectories. This procedure generates a summary of the distinct types of motion that exist across the Earth-Moon system.

Author: Alexandru Caliman
University of Namur, Department of Mathematics
Title: *"Stability of planetary systems in the era of Machine Learning. Application to a formation-driven synthetic population"*

Abstract. The stability of compact three-planet systems is a complex problem that has traditionally been addressed using analytical criteria and powerful tools known as chaos indicators. Here, we show how a Machine Learning (ML) framework can be used to improve the performance of chaos indicators. The ML framework is fed with information derived from three chaos indicators – one variational indicator and two non-variational indicators – as well as several well-known features inspired by planetary dynamics. The performance of our model is evaluated using a dataset of compact planetary systems with a dense network of mean-motion resonances. Furthermore, given the rapid increase in the number of planetary systems detected in recent years, which makes it necessary to study these systems from the perspective of both their formation and their evolution, we apply our model to analyze the stability of hundreds of compact systems generated by the Bern formation and evolution model. This analysis is conducted with the objectives of upcoming space missions dedicated to exoplanets in mind.

This is a joint work with P. Mitra, A.-S. Libert, B. Frenay, S. Marques, R. Eltschinger, Y. Alibert.

Author: Gian Marco Canneori
`Università degli Studi di Torino`
Title: *"The 2-homogeneous 2-centre problem: symbolic dynamics and scattering"*
Abstract. In this talk we consider a planar two-centre problem driven by a singular potential U, which has the following prescribed behaviour close to each centre $c_1, c_2 \in \mathbf{R}^2$

$$U(q) \sim \frac{m_j}{|q - c_j|^2}\,, \qquad m_j > 0\,,\ q \in \mathcal{U}(c_j)\ .$$

We fix a positive value of the mechanical energy and we construct infinitely many periodic solutions in distinct homotopy classes, all avoiding collisions with the centres. We then investigate the existence of invariant sets that are topologically conjugate to the Bernoulli shift. Finally, we analyse the asymptotic behaviour of unbounded trajectories and discuss the existence of scattering solutions with prescribed topological properties.

This is a joint work with Stefano Baranzini and Alberto Boscaggin.

Author: Chiara Caracciolo
University of Padua, Department of Mathematics

Title: *"Existence of KAM tori in the realistic planar three-body problem"*

Abstract. We present a computer-assisted proof of the existence of KAM tori in the planar planetary three-body problem, using physically realistic parameters. The proof is implemented through publicly available software (Locatelli 2023, Mendeley Data) and is based on the construction of the Kolmogorov normal form. We apply this proof to Hamiltonians of the planar planetary three-body problem, after a few preliminary changes of coordinates. The procedure has been tested for different values of the planetary and stellar masses, as well as orbital parameters (semi-major axes and eccentricities), all taken from real observational data of two-planet exoplanetary systems. The proof succeeds for the systems HD11964, HD142, and HD4732, all of which present non-small orbital eccentricities. Finally, we propose a simple criterion to identify planetary systems for which this approach is more likely to succeed.

This is a joint work with Ugo Locatelli.

Author: Josué Cardoso dos Santos
University of Colorado Colorado Springs,
Mechanical and Aerospace Engineering
Title: *"Mean Orbit Propagation with Numerical Recovery of Short-Period Effects in Low-Thrust Perturbed Trajectories for Space Mission Design"*

Abstract. This work presents the modeling and analysis of spacecraft trajectories under a low-thrust acceleration regime using a mean orbital formulation combined with the numerical recovery of first-order short-period effects. The equations of motion are expressed in equinoctial orbital elements, eliminating classical singularities and enabling stable propagation. The proposed approach relies on averaged dynamical models together with near-identity transformations and Fast Fourier Transform techniques to reconstruct short-period perturbations removed during the averaging process. This strategy enables the recovery of key dynamical features of osculating orbits while preserving the computational efficiency of mean-element propagation. The framework incorporates multiple perturbations within a unified formulation, including Earth's gravitational field represented through spherical harmonics, third-body perturbations due to the Moon and the Sun, and low-thrust acceleration effects. Results demonstrate that the proposed methodology achieves high fidelity in reproducing short-period dynamics when compared to fully osculating orbit propagation, while significantly reducing computational cost. The approach is inspired by initial mission design concepts for DESTINY+ mission and is applicable to a wide range of missions in Earth, cislunar, and interplanetary regimes, including cargo transportation architectures supporting sustained lunar and broader space exploration.

Author: Gabriel Antonio Caritá
`Instituto de Geociencias e Ciencias Exatas,`
`Universidade Estadual Paulista (UNESP)`
Title: *"Exploiting Retrograde Mean-Motion Resonances for Spacecraft Trajectory Design in the Bicircular Restricted Four-Body Problem"*
Abstract. This comprehensive study investigates the dynamics, accessibility, and optimal transfer design for retrograde mean-motion resonances, including co-orbital and higher-order configurations, within the Earth-Moon system. Operating under the framework of the Planar Bicircular Restricted Four-Body Problem (PBCR4BP), the research maps libration islands and stability regions, emphasizing the critical influence of solar perturbations and initial solar phase geometry. To access these stable regimes, optimal impulsive transfers originating from Low Earth Orbit (LEO) are designed utilizing advanced metaheuristic optimization techniques. Analyses are conducted to evaluate the robustness of these trajectories against variations in initial conditions and maneuver execution. The results show that retrograde resonant orbits could be used as strategic candidates for lunar logistical gateways, satellite deployment, and intermediate staging for deep-space exploration, indicating the viability of effective, low/modest-cost transfers and pointing out the need for more research on this subject.

This is a joint work with M.H.M. Morais, and A.F.B.A. Prado.

Author: Valerio Carruba
UNESP, Dept. of Mathematics
Title: *"Artificial Intelligence and Machine Learning for Planetary Science in the Rubin Era"*

Abstract. The rapid growth of observational and numerical data is transforming planetary science. Missions such as Gaia and forthcoming facilities, including the Vera C. Rubin Observatory, are producing datasets of unprecedented size and complexity, creating both extraordinary scientific opportunities and significant computational challenges. Traditional approaches based on large-scale numerical integrations and manual analysis are becoming increasingly difficult to apply at the survey scale. Artificial intelligence (AI) and machine learning (ML) are emerging as powerful tools to address these challenges. This plenary lecture reviews recent applications of AI to Solar System small-body research, including resonance identification, asteroid-family classification, clustering of dynamical populations, and automated detection of cometary activity. Deep-learning techniques are enabling pattern recognition in large astronomical datasets, while Physics-Informed Neural Networks (PINNs) combine machine learning with physical laws to accelerate simulations while preserving physical consistency.

The lecture also explores the growing role of large language models (LLMs) in scientific research. Beyond literature exploration and code generation, recent studies suggest that multimodal LLMs may be capable of recognizing dynamical patterns directly from images, which opens new possibilities for scientific discovery and knowledge extraction. While AI offers powerful new capabilities, it also presents important limitations and challenges. Issues such as interpretability, extrapolation beyond training domains, reproducibility, bias, and scientific accountability are discussed, emphasizing the continued importance of physical understanding and human judgment. Finally, a perspective on the future of AI-assisted planetary science in the Rubin era is presented, highlighting the importance of interdisciplinary collaboration and responsible AI practices. As astronomical datasets continue to expand, the central challenge is increasingly shifting from data acquisition to scientific interpretation, making AI an essential component of future discovery.

Author: Anna Maria Cherubini
Department of Mathematics and Physics ‘‘Ennio De Giorgi’’,
University of Salento
Title: *“An application of ergodic theory to the study of space debris”*
Abstract. In recent years, techniques from ergodic theory have been successfully applied to investigate real-world phenomena. In our study, we aim to explore how the spectral properties of transfer operators provide information on the statistical properties of the dynamics of space debris.

A first step to study the motion of a group of small debris fragments, starting from the same position (for example, resulting from an explosion or impact). To do so, we restrict the dynamics to a sphere centred on the Earth, serving as a Poincaré section. The annealed transfer operator associated with the Poincaré map is approximated, via the Ulam method, by a stochastic matrix, defined on a carefully chosen partition of the sphere (constructed using the HEALPix algorithm). Its spectral properties allow us to detect substructures in the Poincaré section, such as subsets of the section that are close to invariant sets under several iterates of the Poincaré map (almost-invariant sets). This approach is first tested on a simple model of a set of space debris in MEO, subject only to the Earths gravitational field and its oblateness.

This is a joint work with Riccardo De Pascalis (Università del Salento) and Gary Froyland (University of New South Wales).

Author: Ariane Courtot
`Institute of Astronomy of the Czech Academy of Science,`
`Department of Interplanetary Matter`
Title: *"What chaos maps teach us about the Taurids meteoroid steam"*
Abstract. Meteoroids are very small bodies ejected from a parent body (comet or asteroid). They are influenced by both gravitational and non-gravitational forces (solar radiation pressure and Poynting-Robertson drag), which makes them interesting to study in terms of chaos. The Northern Taurids (NTA) is a set of meteoroids, which may be coming from comet 2P/Encke, or from a progenitor of 2P/Encke [Egal et al., 2021, 2022]. The uncertainty about the parent body of the NTA makes it necessary to study dynamically. The OFLI is used to draw chaos maps of the NTA. They reveal several dynamical mechanisms, which are in contrast to previous meteoroid streams (Geminids, Draconids, Leonids). In particular, we see the chaotic effect of mean-motion resonances with Jupiter and close encounters with several planets. We also investigated the effect of Lidov-Kozai cycle with Jupiter, and finally how non-gravitational forces affect this picture.

Author: Jérémy Couturier
University of Geneva, Observatory of Geneva
Title: *"Dynamics of zeroth-order three-planet resonances"*

Abstract. While two-planet mean-motion resonances have been extensively studied, the dynamics of three-planet resonances are comparatively less understood, especially when tidal dissipation is involved. We present a one-parameter one-degree-of-freedom pendulum-like model of three-planet zeroth-order resonances of the form pn1 - (p + q) n2 + qn3 = 0 based on a Hamiltonian expansion carried out to second order in the planetary masses. Using the new software Aptidal, we produce stability maps of three-planet zeroth-order resonances, where chaos is measured from the diffusion of the fundamental frequencies. The position of the resonance separatrix predicted by our pendulum-like model accurately matches the regions of mild chaos in our stability maps. In presence of tides, we analytically compute the eigenvalue governing the evolution of the Laplace angle near the 180 degrees equilibrium. We find that its real part is positive regardless of the masses and tidal parameters, demonstrating the instability of three-planet zeroth-order resonances with tides. The inverse of the eigenvalues' real part gives the instability timescale, that is accurately reproduced by simulations of the complete system with Aptidal.

Author: Aurélien Crida
Université Cote d'Azur / Observatoire de la Cote d'Azur,
Nice, UMR7293 Lagrange
Title: *"Planet disc interactions: applications to the Nice model and the reflex instability"*

Abstract. The solar system giant planets have widely separated, non-resonant orbits, with low but non zero eccentricities and inclinations. It is now commonly accepted that this configuration originates in a global dynamical instability which occurred after the gaseous proto-planetary disk was dissipated, referred to as "the Nice model". However, the detailed history of their migration starts in the gas disk; the Grand Tack scenario suggests that Jupiter migrated inwards down to 1.5 AU before being caught in 3:2 MMR with Saturn and the pair migrates back outwards. Besides possibly sculpting the inner Solar system, the outcome of the gas-driven migration is critical for the Nice model as it represents its initial conditions. In this presentation, I will show recent results on the migration of pairs (or more) of giant planets in low-viscosity proto-planetary disks and how these results impact the Grand-Tack scenario and yield a new view of the Nice model (e.g. Jupiter and Saturn start in the 2:1 MMR, not 3:2).

I will also describe a phenomenon which appears in long-term simulations of gas disks due to the so-called indirect term (that is the acceleration of the star due to the asymmetries in the disc). A positive retro-action loop leads to the exponential growth of an $m = 1$ mode in the gas density. It has perturbed a few of our simulations of the migration of the solar system giant planets and can lead to complete destruction of the gas disk. The characterization and applications of this "reflex instability" will be discussed.

Author: Riccardo Daluiso
`Univ. Cote d'Azur, LJAD`
Title: *"Control of Collision Orbits"*
Abstract. In the n-body problem, the forces acting between particles approach infinity when the mutual distances approach zero. Therefore, at collision the dynamics has singularities. Since Levi-Civita and Sundman, the double collision has been regularized, i.e. the singularity has been made to disappear by means of algebraic transformations. Levi-Civita obtained first a regularizing transformation of the Kepler problem based on the inverse of the map from z to z^2 of the complex plane. This conformal map sends the orbits of the harmonic oscillator in the ones of the Kepler problem. The coordinates transformation is coupled with a slowing down of the motion by means of a time change defined by the differential relation dtau = 1/—z—dt. Based on classical results, the purpose of the work is the application of the regularization theory to optimal control. We consider the control of a spacecraft under the attraction of a celestial body, where the control is the thrust, and with the goal of minimizing the physical time. This system exhibits singularities in zero, and thus the theory fails in the study of collision orbits. By applying the Levi-Civita regularization to the controlled system, we obtain an affine system in the new time tau where the vector fields are polynomials, thus analytic. In the reparametrized system, we can broaden the concept of controllability at collision, and apply Pontryagin's principle by allowing orbits that collide or come arbitrarily close to the center of attraction. The extension of the procedure in three dimensions is not straightforward, as it encounters algebraic and topological obstructions. To extend the study to the spatial case, we need to make use of the second Levi-Civita regularization or the KS regularization.

Author: Irene De Blasi
`University of Torino`
Title: *"Study of resonant regimes in Rings' dynamics"*
Abstract. The presence of rings' systems has been observed in many bodies of the solar system, including dwarf planets, asteroids and trans-Neptunian objects. Notably, direct observations on such minor bodies have shown that the rotational frequency of the ring particles around the central body often exhibits a commensurability relationship with the latter's rotational frequency around its axis. In particular, such rings are located in the so-called 1:3 Lindblad resonance.

This talk aims to present a Hamiltonian model, using epicyclic variables, particularly suitable to describe the ring particles' motion close to specific resonances. Using perturbation and bifurcation theory, an analysis of the main resonances of the problem is carried on, with a particular focus on their stability properties, including the possible presence of bifurcations, as well as on the possible presence of KAM tori in their vicinity.

This is a joint work with Alessandra Celletti and Sara Di Ruzza.

Author: Pierfrancesco Di Cintio
`CNR-ISC, INAF-OAA + INFN-Firenze`

Title: *"Rotating radially anistotropic stellar systems: a tale of two instabilities"*

Abstract. We investigate the evolution of a family of sphercial Osipkov-Merritt models with added rotation as function of their anisotropy index and angular momentum J_z. We find that, counterintuitively, rotation has a stabilizing (i.e. mitigating) effect on the strength of the radial orbit instability (ROI) in radially anisotropic models with Fridman-Polyachenko-Shukhman index close to its value for consistency, while slightly enhances the onset of the instability in nearly stable models. Analyzing the growth rate of the models' density modes, we also found that an inner bar is formed even in systems with initial profiles with a central cusp, akin to cored models with rotation and Dejonghe type tangential velocity anisotropy.

Author: Anargyros Dogkas
`University of Pisa`

Title: *"Generalized Le Verrier's development and proper elements of asteroids"*

Abstract. Quasi-integrals of the motion play a fundamental role in celestial mechanics, allowing us to identify dynamical groups through clusterization algorithms [Milani & Knezevic 1990, Wu & Rosengren 2021, Celletti et al. 2021], construct local analytical solutions [Mastroianni et al. 2025], or characterize the stability of the objects under study [Celletti 2023]. Proper elements, in particular, which were first introduced over a century ago [Hirayama 1918], have been the driving factor for the identification of asteroid families. Although this clusterization has been very effective, the analytical theory is limited by the assumptions enforced on the model that is used [Le Verrier 1855, Yuasa 1973]. Additionally, while numerically computed proper elements avoid these limitations [Knezevic & Milani 2000], they also miss the analytical restrictions that define the admissible domains of perturbation theory. In this work, we expand the classical Le Verrier development of the perturbing function, generalizing it to an arbitrary degree of the eccentricity and inclination of the asteroid and the planets, providing explicit expressions for the Hamiltonian of the restricted N-body problem. Using this generalization, we study the dynamical properties of the main-belt asteroids as well as compute analytically higher-degree proper elements, improving the already established results in the literature and providing further details on the admissible domains defined by the dynamical environment of asteroids in the solar system.

This is a joint work with Giulio Baù.

Author: Leonor Cui Domingo Centeno
`Universitat de Barcelona`
Title: *"On the numerical computation of high order approximation of the center manifold for maps"*

Abstract. In this work, we study the asymptotic behavior in the neighborhood of a fixed point under a map. To describe the dynamics near this point, we consider the reduction to the center manifold for a map. Our approach relies on the use of the graph transform method where the invariant center manifold of a fixed point is locally represented as a graph. We numerically compute the parametrization of the center-stable and center-unstable manifolds by using power series expansions around the fixed point. We propose to consider the jet transport technique to obtain an accurate approximation in the expansion of the center manifold through automatic differentiation on the map. The method is considered for an application to celestial mechanics. We are interested in the dynamics near the vicinity of the L3 point of the Earth-Moon system. The model for studying the dynamics of the problem is the Sun-Earth-Moon Bicircular Problem (BCP). To remove the dependence on time in the BCP model, in particular we consider a stroboscopic map defined at time of the perturbation. In this BCP model, the linear dynamics of the periodic orbit that replaces the L3-equilibrium point of the Restricted Three-Body Problem is described by a hyperbolic direction and two elliptical directions. To describe the different types of orbits in the neighborhood of L3, we uncouple the instability behavior caused by the hyperbolic direction from the other elliptical directions. This can be accomplished by performing the reduction to the center manifold around the dynamical substitute of L3. We numerically compute the high order approximation of the center-stable and center-unstable manifolds around this fixed point. As a result, we can then analyze the bounded motion described by the center manifold near the fixed point and visualize the phase space behavior in the original coordinate system.

Author: Nikita Doukhanin
`University of Namur, Mathematics Department, naXys institute`
Title: *"Stability of exoplanet pairs in binary star systems"*

Abstract. Half of all stars are part of multiple systems, most of which are binary stars. More than 10% of the 6,500 discovered exoplanets are found in binary star systems. Understanding the long term stability and dynamics of S-type exoplanets in these systems is essential for detecting them and assessing their habitability. This research focuses on the four-body problem composed of two S-type planets and an external binary companion. Using thousands of N-body simulations, stability maps are achieved with a chaos detector, exploring various binary parameters (eccentricity and inclination). Our goal is to explain the underlying dynamics shaping the stability regions. In particular, we show that the behavior of the planets is strongly affected by resonances, namely the mean-motion resonances (MMR) with the binary companion and/or between the planets themselves, as well as by the von Zeipel-Lidov-Kozai (ZLK) effect for an inclined binary companion. Our study highlights the conditions under which the resonant configuration of a pair of planets can be maintained despite perturbation from the binary companion.

This is a joint work with A.-S. Libert.

Author: Rudolf Dvorak
`Vienna observatory`
Title: *"Jumping planets in the Solar System."*
Abstract. Enlarging the mass of Mars - keeping the other planets masses as they are - an interesting behaviour of Venus and Earth is observed. From the enlargement factor k=5 on these two planets suffer from more or less small regular jumps in semi-major axes. Complete chaotic behaviour occurs from higher values of k on which sooner or later leads to complete instability of the whole system.

Author: Christos Efthymiopoulos
University of Padua
Title: *"Frozen Orbits for Lunar Satellites"*
Abstract. Frozen orbits for lunar satellites are theoretically computed by the secular equilibria of the equations of motion truncated over a generally low-order multipole expansion of the lunar gravitational potential and averaged over short period terms. In reality, many of these theoretical orbits cease to exist when more realistic models of the equations of motion are considered, including a detailed representation of third-body effects and of the short-period effects on the equations of motion. The talk will give a review on the subject along with new results based on our newly developed semi-analytical integrator for lunar satellite orbits (joint results with the SELENA team: M. Tsiganis, I. Gkolias, M. Gaitanas, C. Yanez, and with R.Mastroianni and E. Legnaro).

Author: Szilveszter Fehér
Eotvos University, Institute of Physics
Title: *"Floquet-Theory-based Stability Analysis of the Elliptic Relative Equilibria Solutions of the Gravitational n-Body Problem"*
Abstract. Elliptic relative equilibria solutions are explicit, exact analytical solutions of the gravitational n-body problem, where the bodies travel on elliptic Keplerian orbits while maintaining a self-similar configuration known as a central configuration. A general Floquet-theory-based linear stability analysis formalism of the elliptic relative equilibria solutions is presented and applied to find the linear stability diagrams belonging to the studied central configurations (Lagrangian equilateral triangle, 1+N-gon, and the convex four-body kite configuration). Some other configurations (N-gon, concave four-body kite, rhomboid five-body configuration) were found to be totally unstable for every eccentricity. The linear stability results are compared to the nonlinear simulation of the original n-body equations to see whether the stability criteria for the original nonlinear system are the same as what we get from the Floquet stability analysis.

Author: Sylvio Ferraz Mello
Universidade de Sao Paulo,
Instituto de Astronomia, Geofisica e Ciencias Atmosfericas
Title: *"Mutual perturbations in compact planetary systems"*
Abstract. Classical perturbation theory may be used to compute the main mutual perturbations of planets in compact systems. Analytical expressions exist that allow us to compute the constant perturbation of the semi-major axis and period, the secular dynamics, the forced eccentricities, and the great inequality in longitude. They may be used to predict some observable effects. As examples, we consider the compact systems HD 40307 and L98-59, and the planets of the pulsar PSR B1257+12. The pulsar planets were the only ones for which the precision of the observations (pulsar timings) was large enough to detect periodic perturbations in longitude. These measurements allowed an estimate of the pulsar planets' masses and, by comparison with the minimum masses derived from the usual orbit determination, the estimate of the inclination of their orbits. In the same way, the observed eccentricities of planets in close, compact systems may be compared with the predicted forced eccentricities to give independent estimates of the planet masses, if the damping of the proper eccentricities by tidal friction is taken into account. The comparison of the observed eccentricities with the forced eccentricities predicted by different models may indicate the existence of missing planets and also be used to investigate the reality of planet candidates in search of confirmation.

Author: Alessandra Ferraz da Silva Ferreira
FEG - Sao Paulo State University (UNESP)

Title: *"Orbital configurations of a hypothetical satellite of asteroid (13351) Leucus"*

Abstract. Studies indicate that a significant fraction of Jupiter's Trojans may exhibit characteristics consistent with binary configurations or with evolutionary states derived from originally binary systems [Pravec and Harris (2007), Merline et al. (2002)]. Kalup (2021) analyzed the photometric properties of the Trojans and identified binary asteroid candidates based on the amplitude of the light curve, the rotation period, and estimates of the binary separation parameter. In this analysis, the asteroid Leucus is identified as one of the objects of interest due to its long rotation period (≈ 445.683 h) and the amplitude of the light curve, which are consistent with the hypothesis of a dissociated binary system, as proposed by Nesvorny et al (2020). Although preliminary stellar occultation results suggest that Leucus has an irregular shape and does not provide direct evidence of a nearby satellite, these observations do not rule out the existence of a distant companion.

In this context, the objective of this work is to map the orbital conditions that may characterize, or have characterized, the asteroid Leucus as a binary system at some stage of its evolution.

For modeling the gravitational field of Leucus, a simplified model of the asteroid's shape will be adopted. The potential will be expanded in a series of spherical harmonics up to degree and order 4, whose coeffcients will be obtained from a nonlinear optimization method. This method derives the gravitational coefficients of minor irregular bodies directly from their polyhedral shape models.

Then, the Grid Search Method will be employed to identify symmetrical periodic planar orbits around the asteroid, which, as established in the literature, correspond to closed trajectories in the inertial frame of reference. Additionally, variations will be made to the rotation period of Leucus and the mass of the hypothetical satellite, allowing the classiffication of the obtained orbits according to their geometry (circular, quasi-circular, or elliptical), dynamical regime (synchronous or doubly synchronous), direction of motion (prograde or retrograde), and stability (stable or unstable). Therefore, maps of these possible orbital configurations will be presented.

Author: Yeva Gevorgyan
`KAUST`
Title: *"Synchronisation of the Pluto-Charon binary by inward tidal migration"*
Abstract. It is usually taken for granted that mutual synchronisation of a tidal two-body system is attained through tidal recession, assuming the reduced Hill sphere is not reached. However, synchronisation can be achieved also via tidal approach, provided the Roche limit is not crossed. For each of the two scenarios, we derive the condition under which the evolving synchronicity radius catches up with the tidally evolving orbit. We consider the two scenarios for the Pluto-Charon system and examine the impact-origin hypothesis of Charon's formation against capture. Based on geophysical evidence, we propose that capture appears more likely. Motivated by this conclusion, we investigate both analytically and numerically the capture scenario, wherein the orbital evolution of Charon starts at a higher altitude than present and undergoes tidal descent. We also consider the possibility that Pluto's initial prograde spin underwent a reversal by a tidally approaching retrograde Charon. Depending on the initial conditions, we observe temporary locking of Charon into higher spin-orbit resonances (3:2 to 7:2) during the first 0.5 Myr of the system's evolution. Owing to a greater initial separation between the partners, the power dissipated in each of them turns out to be much lower than in the case of tidal recession of bodies of the same internal structure. The greater initial separation also results in lower tidal stress, which may explain the absence of tidally generated fracture patterns.

Author: Joan Gimeno
`Universitat de Barcelona`
Title: *"Normal Form of the Regularized Primary in the Spatial CR3BP"*
Abstract. I present a constructive methodology to compute the normal form of the dynamics near the regularized primary in the spatial circular restricted three-body problem (CR3BP), using KS variables and normal-form strategy.

Starting from the regularized model, I combine high-order automatic differentiation, recursive homological equation solving to study dynamics around the primary. The resulting normal form provides a clean local representation of the flow close to the collision-regularized region, with explicit control of resonant terms and transport-relevant nonlinear couplings.

This is useful because it yields a numerically stable and geometrically interpretable reduced model in a strongly nonlinear regime where direct coordinates are singular or poorly conditioned. In practice, it enables efficient computation of local invariant manifolds, improved transfer design, and systematic continuation across energy levels. A meaningful application can be low-energy mission design in the Earth-Moon system, including manifold-guided transfer trajectories and sail-assisted transport strategies near libration structures.

The methodology can be general: once a system admits a suitable regularization and a reducible/quasi-reducible invariant structure, the same recursive normal-form pipeline can be ported to other multi-body or perturbed Hamiltonian settings with minor problem-dependent adaptations.

Author: Mar Giralt
LTE - Observatoire de Paris
Title: *"On the Arnold diffusion mechanism in Medium Earth Orbit"*
Abstract. Motivated by the need of preserving the operational orbital regions around the Earth, natural perturbations can be exploited to lead the satellites towards an atmospheric reentry at the end of life. In this way, it is possible to dilute the collision probability in the long term and reduce the disposal cost. In the case of the Medium Earth Orbit (MEO) region, home of the navigation satellites (like GPS and Galileo), the main driver is the third-body perturbation.

In this work, we show how an Arnold diffusion mechanism can trigger the eccentricity growth in MEO, so that the pericenter altitude drops into the atmospheric drag domain. Focusing on the case of Galileo, we consider a hierarchy of Hamiltonian models, assuming that the main perturbations on the motion of the spacecraft are the oblateness of the Earth and the gravitational attraction of the Moon.

First, the Moon is assumed to lay on the ecliptic plane and periodic orbits and associated stable and unstable invariant manifolds are computed for various energy levels, in the neighborhood of a given resonance. Along each invariant manifold, the eccentricity increases naturally, achieving its maximum at the first intersection between them. This growth is, however, not sufficient to achieve reentry. By moving to a more realistic model, where the inclination of the Moon is taken into account, the problem becomes non-autonomous and the satellite is able to move along different energy levels. Under the ansatz of transversality of the stable and unstable manifolds in the autonomous case, checked numerically, Poincaré-Melnikov techniques are applied to show how the Arnold diffusion can be attained, by constructing a sequence of homoclinic orbits that connect invariant tori at different energy levels on the normally hyperbolic invariant manifold.

This is a joint work with E.M Alessi, I. Baldomá and M. Guardia.

Author: Ioannis Gkolias
`Aristotle University of Thessaloniki, Department of Physics`
Title: *"The Role of Resonances in the Post-DART Impact Dynamics of the Didymos–Dimorphos System"*

Abstract. The DART spacecraft impacted the secondary component of the binary asteroid system Didymos–Dimorphos, inducing a 33-minute reduction in the mutual orbital period. Assuming the system was initially locked in a 1:1 spinorbit resonance, the post-impact evolution may exhibit rich dynamical behaviour in the rotation of Dimorphos, ranging from bounded libration at fixed amplitude to chaotic tumbling about all three principal body axes. The range of possible outcomes is strongly dependent on the secondary's shape, with the dynamics governed by nonlinear resonances among the fundamental frequencies of the problem. In anticipation of the arrival of ESA's Hera mission at the system in late 2026, we examine the plausible end states of the system, characterize the role of resonances in its dynamical evolution, and quantify the likelihood of finding the system in a chaotic state.

Author: Clara Grassi
University of Pisa, Department of Mathematics
Title: *"Orbit determination procedure on large datasets"*
Abstract. The Isolated Tracklet File (ITF), managed by the MPC, is a long list of tracklets that have not been assigned to any known object. Large sets of observations, such as the ITF, can be a great resource for orbit determination, both for discovering new objects and for attributing new observations to already known objects. To deal with the large amount of data, the development of efficient methods is crucial. We present a procedure that is able to determine preliminary orbits from large datasets and build on those step by step to obtain refined orbits.

We apply the procedure to sets of real observations of known asteroids and present the results. Test examples using observations from the ITF are also shown.

The same procedure could also be useful to look for potential interstellar objects. Preliminary results of tests on hyperbolic orbits are presented.

This is a joint work with Giovanni F. Gronchi.

Author: Guido Grossi

Department of Aerospace Science and Technology, Politecnico di Milano

Title: *"An Adaptive Collocation Method to Explore Families of Invariant Tori in Astrodynamics"*

Abstract. Invariant tori are among the fundamental structures of many astrodynamical models and play a crucial role in trajectory design. Although numerous approaches exist to compute such manifolds, several challenges remain for numerical schemes. Besides the need for a suitable discretization of the torus, another major difficulty is the rapid growth in the number of unknowns when dealing with high-dimensional tori. This increase can significantly raise the computational cost and hinder convergence.

The method presented in this talk addresses these issues through a two-layer strategy that integrates adaptive collocation with variable spectral resolution to compute invariant curves of a stroboscopic map. The scheme automatically taylors the discretization to the geometry of the torus by locally refining or coarsening the approximation.

When combined with pseudo-arclength continuation, the approach enables efficient exploration of families of invariant tori within the same model (internal continuation) as well as across different models (external continuation) of increasing complexity, from autonomous systems to quasi-periodically perturbed ones.

Author: Alessia Francesca Guido
Università di Trento
Title: *"Effective stability estimates close to resonances with applications to rotational dynamics"*

Abstract. For near-integrable Hamiltonian systems, Nekhoroshev-like estimates provide action confinement for exponentially long times under certain conditions that allow some freedom in parameter choice. We develop an optimization algorithm for choosing parameters that maximize the stability time in the non-resonant case. To further improve the stability estimates, we use perturbation theory to reduce the norm of the perturbing function. We study stability estimates in the neighborhood of resonances by defining sequences of Diophantine frequencies converging to resonant frequencies, for both 1D and 2D non-autonomous Hamiltonian systems. Applied to spin-orbit and spin-spin-orbit models, we show stability results for orbits close to the main resonances associated with such models. This is a joint work with Alessandra Celletti and Anargyros Dogkas

Author: Massimiliano Guzzo
`University of Padova, Dipartimento di Matematica "Tullio Levi-Civita"`
Title: *"On the exponential estimates of the time of stability of elliptic equilibria in Hamiltonian systems: from Nekhoroshev theorems to computer assisted proofs"*

Abstract. After the 1977 paper by N.N. Nekhoroshev about the exponential estimates of the time of stability of the action variables in quasi-integable Hamiltonian systems, we have today a bunch of extensions to the case of stability of elliptic equilibria which are formulated with specific assumptions on the Birkhoff normal forms of a given order. The flexibility in the hypotheses of these results is well suited for applications to Celestial Mechanics. Efficient computations in this context had been obtained with computer assisted implementations of specific steps of the proofs. This talk presents a recent stability result with direct implications for computer-assisted estimates of the stability times.

Author: Nader Haghighipour
PSI/IfA-Hawaii
Title: *"On how the mutual interactions of giant planets damp their secular resonances "*

Abstract. It is widely accepted that the secular resonances of giant planets have played fundamental roles in the formation and dynamical evolution of the inner Solar System. These resonances have shaped the architecture of the asteroid belt and have had significant contributions to the final mass, composition, and orbital architecture of terrestrial planets. It has also been demonstrated that despite its larger mass, the secular resonance of Jupiter is weaker than that of Saturn. While Saturn's secular resonance is the main reason that the inner asteroid belt is almost empty and the terrestrial planets are confined to the region inside 2.1 (au), Jupiter's secular resonance at 0.9 (au) has only a moderate effect on the evolution of the small bodies in its surrounding. Furthermore, recent studies of planetary systems in binary stars have shown that the perturbation of the secondary star strongly damps the secular resonances of giant planets. These findings motivated us to investigate how the mutual interactions of giant planets affect their secular resonances. To that goal, we have developed the full theory of secular resonances in systems with multiple giant planets, and have carried out extensive numerical simulations of their effects on the evolution of protoplanetary disks. We have found that in systems with two and more giant planets, the perturbation of the outer planet has a strong damping effect on the secular resonances of the inner ones. When this planet is replaced by a star, the perturbation of this object becomes so strong that the secular resonances of all inner giant planets are almost completely damped. We have also found that this damping of secular resonances expands the region of terrestrial planet formation in systems with multiple giant planets. In this talk, we present the details of our theory and the analysis of its results. We also discuss the implications of the results, especially the damping of secular resonances, for the formation of terrestrial planets in systems with multiple giant planets.

Author: Marc Jorba-Cuscó
`Universitat Politecnica de Catalunya. Math Department.`

Title: *"On Quasi-Bicircular Solutions to the general planar three body problem"*

Abstract. Quasi-Bicircular Solutions (QBS) to the Planar Three Body Problem (PTBP) where described by Poincaré. These relative periodic solutions to the PTBP can be shown to exist by analytic continuation of a pair of uncoupled Keplerian orbits when ignoring the gravitational interaction between two of the masses (for instance, if two bodies form a binary and the third one is far away from them). Poincaré classified there orbits as of first kind (no inclination, small eccentricity). Those orbits have gained interest in more recent years to use them as basis to build periodically time-dependent restricted models for a fourth massless body. Some numerical investigations of these were conducted in the seventies by Broucke and collaborators. In this work we numerically trace the family in several cases (for several choices of the mass-parameters and several initial conditions). We explore the stability of the family as well as their limits. We find several that QBS can be continued to high-eccentricity orbits.

This is a joint work with Joan Gimeno, Begoña Nicolás and Ángel Jorba.

Author: Zoran Knezevic
Serbian Academy of Sciences and Arts
Title: *"On the catalogs of asteroid synthetic proper elements: comparison of the contents and data accuracy"*

Abstract. The asteroid proper element catalogs nowadays typically contain data sets for well over a million asteroids, and are available from three different sources (AstDyS, Asteroid Families Portal, PDS A&D subnode). For the sake of the users we present here an overview of the contents of these catalogs, with particular emphasis on the Trojan asteroids, Trans Neptunian Objects and resonant populations, and discuss the accuracy of the data in terms of the most common appplications of proper elements: classification of asteroids into families and study of the asteroid long term dynamics. We consider methods and procedures to compute proper elements and frequencies, and compare their values as well as their errors. Finally, we provide some recommendations on the choice and use of proper elements in practice.

Author: Barak Kol
Hebrew University of Jerusalem
Title: *"Reformulation and flux-based statistical theory of the egalitarian three-body system"*

Abstract. This talk addresses the egalitarian (non-hierarchical) Newtonian three-body system, which is known to be chaotic. It will present a novel formulation of the dynamics, a validated statistical theory, and several recent advances.

The reformulation introduces a set of dynamical variables: a complex relative position vector, a bi-complex number and a 3d triangle-geometry space. The statistical theory departs from the foundations of previous approaches and is instead based on the flux of phase-space volume. It involves a leaky container analogy with a pipe-joint geometry.

Numerical simulations show detailed agreement with a leap in precision, establishing the flux-based theory as the most accurate statistical theory of the system to date. Recent advances include the double pendulum as a leading example of the flux-based method. Along the way, these ideas will be placed in their broader historical context, tracing relevant developments across several centuries.

Author: Tamas Kovacs
Eotvos University, Institute of Physics
Title: *"Detecting Hamiltonian chaos from scalar time series via recurrence plot divergence"*

Abstract. We present a study demonstrating that the recurrence-based divergence quantifier (DIV), traditionally applied to dissipative systems, serves as an effective finite-time chaos indicator for conservative and Hamiltonian dynamics. DIV is defined as the inverse of the longest diagonal line in a recurrence plot and can be computed directly from an observed time series, requiring no knowledge of the underlying equations of motion. Benchmarking against the well-established fast Lyapunov indicator (FLI) on the standard map, we find agreement probabilities exceeding 95 across a broad range of nonlinearity parameters. A key result of our study is the identification of distinct power-law decay rates of the spatial average, depending on the dynamical nature of the orbit: regular trajectories exhibit a decay scaling of 1/N, mirroring the asymptotic decay of the maximal Lyapunov exponent, whereas chaotic trajectories decay significantly more slowly. These power laws are robust to the choice of recurrence plot parameters, including the norm and recurrence rate. Crucially, equivalent scaling laws are recovered when DIV is computed from scalar observables alone, both with and without delay-coordinate phase space reconstruction, broadening the method's applicability to real observational data. The results generalize beyond the standard map to Poincaré sections of resonant Hamiltonian models, including overlapping resonances and a modulated pendulum. DIV also proves sensitive to sticky dynamics, where temporary phase-space trapping produces characteristic staircase signatures distinguishable from purely regular or chaotic behavior. The computational complexity of DIV is $O(N^2)$, placing it among orbit-based diagnostics that avoid the tangent-map computations required by variational methods. These findings establish DIV as a theoretically grounded and practically viable tool for chaos detection in conservative systems directly from time series.

Author: Sacha Kuhn
Observatoire de la Cote d'Azur
Title: *"Does secular chaos matter in super Earth systems?"*
Abstract. Recent studies on the Solar System have shown that the ergodic assumption is not verified for the secular chaos, with quantities being almost conserved over planetary systems' lifetime. In particular, the total inclination angular momentum deficit (AMD), which describes the average mutual inclination of the orbits is almost conserved. This is an unexpected result since the variables are strongly influencing each other and exchanges occur for very inclined systems (e.g. Kozai-Lidov oscillations). We analyze the reasons for this quasi-conservation and assess its validity for typical exoplanet system configurations, in particular super Earth systems. This conservation brings into question the real impact of secular chaotic dynamics on the longterm evolution of such systems. Work in collaboratoration with Antoine Petit, Federico Mogavero

Author: Bhanu Kumar
Department of Mathematics, University of Michigan
Title: *"On the Use and Structure of Unstable Resonant Orbits for Saturn Tour Design"*

Abstract. The unstable orbits contained within mean motion resonances are of crucial importance for mission design in outer planet moon systems, given their ability to help facilitate propellant-free changes of the spacecraft orbit's semi-major axis. While preliminary design of such missions has traditionally relied on patched-conic or restricted 3-body problem models – which include the gravity of at most one moon at a time – both of these models neglect the often-significant perturbations from other, "untargeted" moons on these resonant orbits. This can lead to difficulties when ephemeris transition is then attempted. And in the Saturnian case, even the more basic step of understanding such resonant pathways in a restricted 3-body model has been lacking, with previous studies focusing on such trajectories only in patched conics.

Given current scientific recommendations to both ESA and NASA for future missions to Saturn's moon Enceladus, as well as the above-mentioned gaps, a higher-fidelity, multi-body understanding of unstable resonant orbits in the Saturn system would be of use. As a rst step in this direction, this study focuses on characterizing unstable orbits resonant with Titan and Rhea – with which any Enceladus mission must initially carry out flybys – using 3- and 4-body models. We start by analyzing unstable resonant orbits and the heteroclinic connections they induce in the planar circular restricted 3-body problem (PCRTBP) models for Titan and Rhea separately. We show that at the higher energies usually used for Saturn moon tours, there is a natural correspondence between these PCRTBP trajectories and those from patched conics. Then, using a Saturn-Titan-Rhea restricted 4-body model, we study the perturbative e ect of each moon on the PCRTBP unstable resonant orbits of the other – the same orbits which also correspond to those of interest from past patched-conics studies. By studying the persistence of tori as well as the perturbation-induced generation of secondary resonances inside normally hyperbolic invariant manifolds at mean motion resonances, we determine regions of similarity as well as of significant dynamical di erences vs. the patched-conics and PCRTBP models.

Author: Eduard Kuznetsov
Ural Feder. Univ., Inst. of Nat. Sc. and Math.,
Dept. of Astron., Geodesy, Ecol., Env. Monitoring
Title: *"Evolution of the compact planetary system K2-72"*
Abstract. We investigate the dynamic evolution of the compact four-planet system K2-72, which orbits an M-type dwarf star. The system consists of three Earth-sized planets and one super-Earth. We searched for low-order mean-motion resonances within the uncertainties of the planets' orbital periods. Several evolutionary scenarios for the K2-72 system over a timescale of 100 Myr were explored using the Posidonius software, which accounts for tidal interactions. Our results indicate that the compact planetary system K2-72 is likely to have evolved away from low-order resonances. Nevertheless, the orbital semi-major axes exhibit significant secular evolution driven by tidal effects. We estimated the drift rates of the planets' semi-major axes and found that approximately 120-150 Myr ago the three inner planets may have been close to resonance, with K2-72b and K2-72d near a 4:3 resonance and K2-72d and K2-72c near a 2:1 resonance. Finally, we searched for initial conditions that could explain the transition from a past resonant configuration to the currently observed non-resonant state. The work was supported by the Ministry of Science and Higher Education of the Russian Federation, project FEUZ-2025-0003.

Author: Andrew Langford
Purdue University, School of Aeronautics and Astronautics
Title: *"Periodic Orbit Centered Normal Forms in the Circular Restricted Three-Body Problem"*

Abstract. Periodic orbits play a distinguished role in organizing the phase space structure of non-integrable Hamiltonian systems. Normal form theory complements this perspective by decomposing local dynamics into separable components via symplectic transformations. In the Circular Restricted Three-Body Problem (CR3BP), normal forms centered on libration points yield rich characterizations of transit motion and resonance interactions near these equilibria. However, such equilibrium-centered normal forms are valid only within the convergence region of the truncated dynamics - a narrow subset of the full dynamical structure present in the CR3BP.

This work describes the construction of normal Hamiltonian systems centered on periodic orbits in the CR3BP. Constructing normal form centered on a periodic orbit rather than an equilibrium point substantially expands the domain over which these techniques can approximate the local dynamics. The approach presented builds on previous periodic orbit normal form contributions and adopts a Lie-Floquet framework. The approach decomposes the invariant subspaces of the flow relative to the periodic orbit and normalizes coordinates through a Lie-Deprit procedure. The result is a local, integrable coordinate description of the ambient Hamiltonian flow in the neighborhood of a chosen periodic orbit. Applications to internal resonances and canonical perturbation theory are also included.

Author: Jacques Laskar
LTE/Observatoire de Paris
Title: *"Recovering the orbital motion of the planets in the solar system through sedimentary geological records"*

Abstract. To recover the past evolution of planetary motion in the Solar System, we can build the most precise model possible, taking into account all small effects that affect planetary trajectories, and adjust this model to all available observations, both from Earth and from space. This is the principle behind planetary ephemerides such as INPOP or DE. The accuracy of these models is determined by comparison with the available observations, covering a few centuries. However, to understand the past evolution of planetary motions over millions of years (Ma), the long-term propagation of such solutions has intrinsic limitations.

Indeed, a first obstacle is that it is difficult to assert that the Solar System has not been affected by external phenomena over extended timescales, such as passing stars. Moreover, planetary motion is chaotic (Laskar, 1989), and the exponential divergence of the orbits practically limits any deterministic prediction to about 60 Ma.

To go beyond this horizon of predictability, additional information is required. I will show here how such information can be retrieved from geological sedimentary data.

Variations in Earth's orbit and axial tilt induce climatic changes on its surface, which are recorded in sedimentary deposits. These are the so-called Milankovitch cycles. The problem is that this record is extremely noisy, full of unconformities, and expressed in terms of depth rather than time. A critical aspect of this analysis is thus the estimation of the sedimentation rate, which determines the timedepth transfer function relating geological depth to time.

Within the AstroGeo project, we have devised a method to establish a continuous timedepth transfer function throughout the record, accommodating variable sedimentation rates, and to extract the primary astronomical signal from the geological sequence. This is achieved using a genetic algorithm that adapts to a wide range of sedimentation rate variations. This statistical approach enables the reconstruction of an astronomical signal (e.g., eccentricity and/or precession) purely from the stratigraphic sequence.

This opens the possibility of following the orbital evolution of the Solar System in the remote past, beyond the horizon of predictability imposed by the laws of celestial mechanics.

Author: Matheus Lazarotto
Instituto de Matematica e Estatistica,
Universidade de Sao Paulo (IME-USP)
Title: *"An angular-momentum preserving dissipative model for the point-mass N-body problem"*
Abstract. A simple model emulating energy dissipation due to tidal effects is proposed. In it, forces acting between masses remove energy but preserve the total angular momentum of the system. We study the effect of such forces on the particular family of orbits in central configurations, and show that a specific dependence on the mutual distances between the bodies leads to homographic equations equivalent to those of the two-body problem with dissipation. We then describe in detail the topology of solutions of the dissipative two-body system via Poincaré compactification.

Author: Eduardo Leandro
`Department of Mathematics, Federal University of Pernambuco`

Title: *"Moments and Geometric Equilibrium Equations"*

Abstract. We review the classical theory of moments, and apply it to formulate problems of equilibrium configurations of systems of interacting particles under the sole hypothesis that the interactions are between pairs of particles and are directed along the lines connecting the pairs. Complying external forces are admitted, so the description of some dynamical equilibrium configurations, such as relative equilibria in Celestial Mechanics, is included in our approach. Moments provide a unified framework for equilibrium problems for an arbitrary number of particles in arbitrary dimensions. The equilibrium equations are homogeneous and invariant by oriented isometries. Celebrated equations from Celestial Mechanics such as Albouy-Chenciner and Dziobek equations for n-body central configurations, as well as novel families of equilibrium equations, are obtained through simple algebraic procedures which require neither reduction by isometries nor a variational principle for their determination. For the sake of concreteness, we present some new sets of central configuration equations.

Author: Walther Litteri
University of Strathclyde, Aerospace Centre of Excellence
Title: *"A Physics Informed Fourier-Chebyshev Generator of Trajectories in the Restricted Three-Body Dynamics"*

Abstract. This work proposes an Artificial Intelligence model for the generation of trajectories in the Restricted Three-Body Problem. A unified representation combining a Fast Fourier Transform decomposition of the trajectory with a Chebyshev parameterisation of the residual is first introduced and validated on periodic, quasi-periodic orbits, and heteroclinic connections. A Normalising Flow Model is subsequently trained on the resulting trajectory features with a physics-informed regularisation loss term based on the gradient of the action functional. Unlike conventional physics-informed approaches relying on the numerical integration of the equations of motion during training, the proposed loss can be evaluated directly on the reconstructed trajectories, resulting in a substantially reduced computational cost. The generative performance of the model is then assessed through the same physics-informed metric, demonstrating that the proposed regularisation effectively suppresses the generation of dynamically inconsistent trajectories and improves the overall physical quality of the generated samples. Furthermore, when combined with a class-aware conditional sampling strategy, the model successfully generates trajectories consistent with the target dynamical classes, including periodic orbits, quasi-periodic trajectories, and natural transfer solutions. This behaviour reflects the highly structured representation learned by the model and its capability to capture the underlying organisation of the phase space. Beyond the specific application considered here, the proposed framework provides a general methodology for the probabilistic modelling of trajectories in celestial mechanics and, more broadly, in nonlinear dynamical systems.

Author: Federico Mogavero
Università di Padova - Dipartimento di Matematica
Title: *"A chaotic yet stable model of the inner Solar System"*

Abstract. The existence of quasi-integrals of motion in the gigayear-timescale dynamics of the inner planets of the Solar System was recently revealed through a hybrid approach combining numerical integration and perturbative techniques implemented via computer algebra. Numerical evidence suggests that these slowly-varying variables curb the excursions of Mercury toward high eccentricities, which eventually lead to instability. We investigate the constraints that the quasi-integrals exert on orbital evolution by deriving a dynamical model in which they are exactly conserved. Surprisingly, this simplified yet complex dynamics reveals the same level of chaos as the nominal system, but exhibits no instability. Employing techniques from statistical mechanics, we sample the microcanonical density of states associated with the new model to reveal its phase-space structure.

Author: Kenta Oshima
`Suwa University of Science,`
`Department of Mechanical and Electrical Engineering`
Title: *"Novel Barrier Structures in the Circular Restricted Three-Body Problem"*

Abstract. The zero velocity curve (ZVC) and the zero velocity surface (ZVS) are well-known examples of barrier structures that separate the regions of possible and impossible motion from an energetic point of view. After generalizing the regions of possible and impossible motion, I will introduce two different types of novel barrier structures in the circular restricted three-body problem. The first type, the associated state of which globally minimizes the energy function, regulates the motion of a particle complementarily to the ZVS. I will highlight its feature along with the comparison with the ZVS. The second type, the associated state of which locally minimizes the energy function, also plays the role of a barrier but in a conditional manner. Significantly, it robustly exists and regulates the motion even in the high-energy regime, where the ZVC ceases to exist.

This presentation will be based on the two papers: 1) "A hidden barrier surface complementary to the zero velocity surface in the circular restricted three-body problem", Oshima, CMDA, 2024, 2) "On the concept of conditional barrier structures in the planar circular restricted three-body problem", Oshima, CMDA, 2025.

Author: Luke Peterson
University of Texas at Austin,
Department of Aerospace Engineering and Engineering Mechanics
Title: *"Effective Stability of Near-Rectilinear Halo Orbits in the Earth-Moon System"*
Abstract. Near-rectilinear halo orbits (NRHOs) around Earth-Moon L2 in the Circular Restricted 3-Body Problem (CR3BP) exhibit a complex dynamical landscape, featuring a band of normally elliptic orbits embedded within regions of strong instability. This coexistence of stable and unstable dynamics, amplified by the numerical sensitivity associated with close lunar passages, makes the long-term behavior of trajectories near NRHOs a delicate and intrinsically nonlinear problem. Understanding the effective stability of these elliptic orbits is therefore a critical challenge, lying at the intersection of local normal form theory and global instability mechanisms.

To quantify finite-time confinement, we formulate a rigorous framework for effective stability using discrete Poincaré maps. By employing jet transport to compute high-order Taylor expansions, we construct explicit polynomial normal forms. We derive discrete Nekhoroshev-type estimates by identifying the optimal normalization order, which balances the asymptotic convergence of the map's analyticity domain against the cumulative penalty of low-order small divisors.

Applying this framework to the Earth-Moon system, we map the resulting geometric limits directly into physical spatial coordinates. Crucially, we demonstrate that for practical mission lifetimes (e.g., 10-50 years), the required stability is vastly shorter than the characteristic Nekhoroshev accumulation time. Consequently, the effective stability region is not constrained by time-dependent exponential drift, but is instead governed entirely by the maximum analytical domain of the optimized normal form. These derived spatial envelopes establish explicit geometric boundaries for the intrinsic local stability of elliptic NRHOs, providing a rigorous mathematical characterization of their nonlinear confinement within the CR3BP.

Author: Antoine Petit
`Observatoire de la Cote d'Azur`
Title: *"Stellar spin down can lock hot Neptunes in polar orbits"*
Abstract. The growing number of measured 3D spin-orbit misalignement angle for short period exoplanet tend to show a statistical clustering of polar orbits. The favorite mechanism for such misaligned exoplanets is the high eccentricity migration triggered by the von Zeipel-Kozai-Lidov (ZLK) resonance followed by tidal circularisation. Yet, ZLK simulation tend to create a distribution of obliquities with two peaks 20-30 degrees away from the polar orbits rather than centered on the polar orbits.

We explore a scenario where a planet migrated through the high-eccentricity migration route due to the insteraction with a distant companion.

We show that as the star spin down, the Laplace radius decreases, and can cross the planet orbit. As the planet nears the Laplace surface, it can get captured in the resonance centered around the orthogonal Laplace equilibrium that lies at an obliquity of $\pi/2$ in the plane orthogonal to the line of node of the perturber. The obliquity end up librating around the polar orbits explaining the observed cluster. As the orthogonal Laplace equilibrium weakly depends on the perturber orbital configuration, this mechanism provides a general explanation for locking in polar orbits. This scenario gives constraints on the time of arrival of the misaligned planets as they need to circularize before the host star has significantly spinned down.

Author: Elke Pilat-Lohinger
`Technical University Graz,`
`Institute of Theoretical and Computational Physics`
Title: *"The effects of binary Star on planets in the habitable zone"*
Abstract. Almost 4 900 exoplanetary systems have been discovered to date; of these, less than 20 % are found in stellar systems, even though the majority of solar-type stars form as binary stars. This raises the question of whether this is a bias of the observation, or whether binary stars suppress the formation of planets. In this context, a brief review of planetary motion in double star systems is first provided, followed by an overview of the binary systems in which exoplanets have been discovered. Based on results of long-term N-body simulations, binary specific perturbations are discussed, and the extent to which they have a negative impact on planetary formation, stability and habitability.

Author: Alexandre Prieur
LTE, Observatoire de Paris-PSL
Title: *"Marchal's family: inclined co-orbitals in the three-body problem"*
Abstract. In the three-body problem, for all values of the masses, the vertical eigenvectors of the Lagrange relative equilibrium give rise to a family of spatial quasi-periodic orbits; these orbits are periodic in a rotating frame. For the equal masses' cases, this is the P12 family with its order-12 symmetry group, which ends at the figure Eight solution (studied in Chenciner-Fejoz-Montgomery (2005), Chenciner-Fejoz (2008) and Calleja, Garcia-Azpeitia, Hénot, Lessard and Mireles (2024)). In 2009, Christian Marchal studied this family in the special case of the average restricted secular problem. We propose a numerical study to follow this family in the general case. We show that the family discovered by Marchal exists in the full three-body problem (neither restricted nor secular) for a wide range of masses, and that the stability of its orbits evolves along the family, sometimes leading to stable systems for masses exceeding the Gascheau's value. We also link this family to P12: in the equal masses case, they are identical up to a bifurcation, where P12 continues towards the Eight and the Marchal branch goes to collisions.

Author: Alexander Prokopenya
`Warsaw Univ. of Life Sciences, Inst. of Information Technology`
Title: *"Perturbative Methods in Studying the Secular Evolution of a Planetary System of Many Bodies with Variable Masses"*

Abstract. A classical $(n+1)$-body problem with n planets moving around a central star of variable mass on quasi-elliptic orbits is considered. The bodies are assumed to attract each other according to Newton's law of universal gravitation. The star and planets may change their masses anisotropically and this leads to the appearance of reactive forces. The problem is not integrable even in the case of constant masses and so the perturbation theory based on an exact solution to the two-body problem is usually applied to investigate the dynamics of planetary system. Note that in the case of variable masses the two-body problem is not integrable, in general, but it may be modified and reduced to the form being appropriate for application of the perturbation theory. In the present talk, the problem is analyzed in the framework of Newtonian's formalism and equations of motion are derived in terms of the osculating elements of aperiodic motion on quasi-conic sections. Special attention is paid to the practically important case of small eccentricities and inclinations, when the perturbing forces may be replaced by the corresponding power series expansions. The differential equations of the perturbed motion are averaged over the mean longitudes of the planets and the evolutionary equations describing behavior of the orbital elements over long periods of time are derived. We solve the evolution equations numerically and demonstrate that the masses variation can modify essentially the evolution of the system.

This is a joint work with M.Zh. Minglibayev and S. Bizhanova.

Author: Giuseppe Pucacco
`Department of Physics - University of Rome Tor Vergata`
Title: *"Stability of Laplace-like resonances"*
Abstract. We describe the generalisation of the de Sitter equilibria in multi-resonant 1+3 body systems in the case of first-order resonances. The prototype of these systems are the Galilean satellites of Jupiter, but several exo-planetary systems are currently observed in these resonances and the catalog count is growing. However, formation models predict higher occurrence of these configurations and observations seem to confirm their disappearance in aged systems. We use the outcomes of the theory to feed numerical simulations to investigate the possible breaking mechanisms of first-order resonant chains.

Author: Philippe Robutel
LTE/Observatoire de Paris-PSL
Title: *"Quasi-periodic co-orbital motions in the planetary three-body problem"*
Abstract. Proving the existence of quasi-periodic orbits (hence stable over infinite times) in the planetary three-body problem (where one mass is significantly larger than the other two) encounters the problem of the degeneracy of the Kepler problem.

In a seminal article, Arnold (1963) succeeded in overcoming this difficulty and proved that, except in a negligible set, all the trajectories in the planar planetary 3-body problem are indeed quasi-periodic. Then, following the ideas of Herman, Féjoz (2004) extended this result to the general case of the spatial planetary n-body problem. Finally, Chierchia and Pinzari (2011) obtained a refined result in this setting by means of a proper symplectic reduction.

Here, we consider the co-orbital 3–body problem, which presents an additional fundamental difficulty, since the two "planets" are in 1:1 orbital resonance (while their periods should be in a Diophantine ratio for the classical KAM theory).

In this context, Leontovich (1962) proved the Kolmogorov nondegeneracy in the restricted three-body problem (TBP) close to L4 and L5, allowing the application of KAM theory to find small quasi-periodic tadpole orbits. Deprit and Deprit–Bartholome (1967) generalized this result without, however, reaching large-amplitude tadpole orbits.

Cors, Palacián, and Yanguas (2019) addressed the problem of horseshoe quasi-periodic orbits using computer-assisted methods. At the same time, Niederman, Pousse, and Robutel (2020) treated the problem within a fully theoretical framework. They rigorously proved, for the 3-body problem, the existence of quasi-periodic horseshoe-type orbits densely filling 2-dimensional elliptic invariant tori (hence non-maximal), with the union of these tori having zero measure.

Using this latter framework coupled with the techniques introduced by Biasco and Chierchia (2023), we prove the existence of a large-measure set of Lagrangian invariant tori supporting quasi-periodic co-orbital motions. This proof applies to both tadpole and horseshoe orbits. Especially, we crucially rely on the reduction of the main part of the Hamiltonian to the standard form introduced by Biasco and Chierchia and the almost sharp estimates on the domain of analyticity in action–angle variables that they provide.

Author: Giovanni Romagnoli

Università di Pisa - Dipartimento di Ingegneria dell'Informazione

Title: *"Angle-only relative orbit determination with the Keplerian integrals of motion"*

Abstract. The increasing number of resident space objects makes complete coverage from ground-based systems increasingly difficult in terms of cost, visibility, and scalability. Space-based observations provide a complementary approach, where an orbiting platform acquires onboard measurements and processes them autonomously. Among the available sensing techniques, optical sensors are the most attractive due to their low power and weight requirements, large operational range, and widespread availability. However, they suffer from an intrinsic ambiguity in range.

We consider the problem of relative orbit determination from angle-only measurements. To estimate the range, existing approaches typically rely on additional information, such as maneuvers, multi-view geometries, or prior assumptions on the target. In this work, a different approach is proposed. We employ the linkage method introduced by Gronchi et al. (CMDA, 2015) for initial orbit determination of asteroids. This method uses the conservation of the Keplerian integrals to find a preliminary orbit from two arcs of ground-based optical observations. This algorithm is applied to measurements acquired onboard and is therefore formulated in a non-inertial reference frame attached to the observer. This leads to a finite set of admissible solutions for the range and range-rate, without requiring an initial guess.

The quality of the candidate solutions is evaluated through some metrics (Rodríguez et al., Icarus, 2022), and the associated covariance is obtained through propagation of the measurement uncertainty. The selected solutions are then refined through a batch estimation using all available observations.

Numerical simulations with realistic measurement errors and perturbed dynamics show convergence of the method and accurate reconstruction of the relative state, with consistent uncertainty estimates, supporting the feasibility of this approach for space-based SSA.

This is a joint work with G. Baù and G. Bucchioni.

Author: Aaron Jay Rosengren
Univ. of California San Diego, Mechanical and Aerospace Engineering
Title: *"The Astrodynamics of Cislunar and Translunar Space"*
Abstract. The Earth-Moon environment is best understood not as a single beyond-GEO region, but as a multiscale circumterrestrial phase space partitioned by changes in perturbation hierarchy, resonance structure, and gateway topology. In this talk I present a set of astro-cartographies of cislunar and translunar space that synthesize orbit-averaged lunisolar theory, semi-analytical resonance atlases, restricted three-body dynamics, and direct numerical mapping into a unified celestial-mechanical picture. Within this framework, cislunar space begins at the Laplace transition, where Earth's oblateness ceases to dominate the long-term precession, and extends outward through a secularly dominated zone characterized by von Zeipel-Lidov-Kozai-type dynamics and lunar apsidal/nodal secular resonances. Farther out, the outer cislunar domain becomes organized by the principal interior lunar mean-motion commensurabilities, before merging into the circumlunar gateway region associated with the Earth-Moon L1/L2 bottlenecks and the topology of the Hill regions.

Beyond the Moon, the translunar realm emerges as a distinct outer circumterrestrial regime in which the Moon acts as an interior perturber while the Sun remains an exterior perturber. I will show how this produces a genuinely mixed secular architecture, with the classical solar exterior Kozai mechanism competing against the complementary inverse-Kozai-type lunar dynamics, and a mixed resonant architecture in which exterior lunar commensurabilities coexist with the first low-order solar resonances. Chaos-indicator and fate-class maps reveal where this outer domain is quietly secular, where it is resonantly structured, and where it becomes a broad transport borderland supporting sticky residence, temporary capture, collision, and escape. The aim is to recover not merely a list of resonances, but a coherent geography of motion in the Earth-Moon system, one that clarifies the dynamical relation between cislunar and translunar space and provides a common spatiographic language for long-term stability, transport, and weakly bound motion.

Co-authors: Shane D. Ross, Anjali Rawat, Bhanu Kumar.

Author: Mattia Rossi
`LTE - Observatoire de Paris`
Title: *"Capture and escape from the 2:1 resonance between Ariel and Umbriel in a fast-migration scenario of the Uranian system"*

Abstract. Recent studies suggest that Uranus may exhibit a higher tidal dissipation than previously assumed. This enhanced dissipation leads to a faster orbital migration of its five major moons: Miranda, Ariel, Umbriel, Titania, and Oberon. In particular, Ariels fast migration implies that the crossing of the 2:1 mean motion resonance with Umbriel is extremely likely, and it could have occurred in recent times (within the last one billion years). Capture into this strong resonance would have induced significant tidal heating within Ariel, possibly explaining its resurfacing. For low initial eccentricities, we confirm that the moons are always captured into this resonance. As the system is not currently involved in any mean motion resonance, we investigated possible dynamical mechanisms for exiting the 2:1 resonance. We show that the resonance could have been broken by a further resonant encounter with Titania. While the crossing of the three-body resonant chain produces a general excitation of the orbital elements of all moons, a fraction of our simulations results in final low eccentricities and inclinations, which can eventually match the current orbital features of the system.

Author: Gabriela Rufino Travassos
Aeronautics Institute of Technology (ITA)
Flight Mechanics Department
Title: *"A Planar Circular Restricted Three Body Model for the Small Bodies Population for HD 159243 system "*

Abstract. Small bodies are essential for understanding the present-day dynamics and the evolutionary pathways that led to the current configurations of our planetary system. While dynamical structures arising from the circular restricted three-body problem (CR3BP) govern key transport mechanisms, such as escape, capture, and the fate of trajectories in subsystems like the Trojan asteroids of Jupiter, the Earth-Moon system, and the transport between the Jovian moons, there exist interplanetary regions of the Solar System where these structures are not the primary drivers of the dynamics. In contrast, CR3BP-like mechanisms may play a more dominant role in less crowded planetary systems, particularly those hosting massive planets in close proximity to their host star. One such example is HD 159243, a solar-type star hosting two Jupiter-like planets at orbital distances of less than one astronomical unit. In this system, the considerable mass ratios between the star and each planet, combined with their compact configuration, suggest a substantial contribution from three-body dynamics. Furthermore, the proximity of the planets may allow for overlapping invariant manifold structures with similar energy values, potentially giving rise to heteroclinic connections that act as transport channels between the primaries' regions. During this presentation we will show a Planar Restricted Three-Body Problem (PCR3BP) model for HD 159243 planetary system serving as an example of other compact two-Jupiter lie planets planetary systems. Our model uses the strategy of the patched-three body to connect invariant manifold tubes of Lyapunov orbits of the system creating transfer channels between the planets and testing the long-term conservation of these orbits.

Author: Daniel Scheeres

`Smead Aerospace Engineering Department, University of Colorado Boulder`

Title: *"Formation of Contact Binaries through the Creation of Asteroid Pairs"*

Abstract. Asteroid pairs are formed from the dynamical evolution of a bound system that eventually leads to the ejection of some of the system components. These situations can occur following a spin fission event, a catastrophic disruption or even during the initial collapse phase of a rubble pile's formation. When such an ejection occurs, it naturally draws energy from the system's remaining bound orbits and their spins, through contact or gravitational interactions, and naturally decreases the energy of the remaining system. The angular momentum and energy budget for such an ejection splits the initial system quantities into a mutually escaping angular momentum and associated escaping energy, which are "lost" to the system, and the remaining angular momentum and energy which is then distributed between the bound systems. If sufficient energy is removed, the remaining system components can be forced into forming contact binaries.

In the Full Body Problem it is possible to give rigorous analytical constraints on the angular momentum and energy of the remaining components as a function of the ejection energy. Through this we can show that for 3-body systems a frequent outcome of ejection is the formation of a contact binary, due to loss of energy in the remaining bound pair. For a simple system, these bounds can be developed as a function of the initial system angular momentum and energy, providing rigorous bounds on the angular momentum of the remaining system. Test numerical simulations will be shown that verify the theory and provide some initial statistics for the frequency of contact binary formation as a function of initial system angular momentum and energy.

Author: Bruno Sicardy
Observatoire de Paris, LTE
Title: *"The rings that should not exist – the challenges of streamline crossing and Roche limit"*

Abstract. Rings are observed around all the giant planets and also around four small objects of the outer solar system. These disks are composed of innumerable colliding particles interacting gravitationally. They obey the laws of Celestial Mechanics, as they should, but their interactions cause in counter-intuitive behaviors such as confinement that is opposite to the expected spreading caused by collisions, or their survival well outside the Roche limit.

The classical Hamiltonian approach satisfactorily describes in the great lines how mean motion resonances perturb a ring. However, collective effects in a ring are an ingredient of paramount importance that requires not only Celestial Mechanics, but also hydrodynamics. Analytical works developed in the 1980's explain how first-order resonances $m/m+1$ may lead to angular momentum flux reversal, and then to confinement. However, the rings recently discovered around small bodies appear to be confined near second-order $m/m+2$ resonances. This is a problematic situation since these resonances cause streamline crossings that should rapidly disperse the ring. N-body simulations actually show that a collisional disk is able to re-distribute the energy it receives from the second-order resonance into "innocuous" free modes of the type $m/m+1$ that in turn lead to confinement. Although this process is observed in nature and in simulations, its analytical explanation is currently not given.

Another paradoxical behavior is the presence of dense rings well beyond the Roche limit of the trans-Neptunian object Quaoar. Once more, nature proves to have more imagination than us. N-body simulations combined with theoretical calculations show that accretion may be prevented outside the Roche limit if the velocity dispersion in the disk remains larger than a critical level. This blocks the phase transition that normally lead to accretion of the ring into a satellite, and thus its disappearance. This behavior calls for a new definition of the Roche limit that is not only static, but also kinematic.

Author: Alan Signor
Sao Paulo State University (UNESP)
Title: *"Resonance capture of Trans-Neptunian objects at arbitrary inclination"*
Abstract. Since the first detection of Trans-Neptunian Objects (TNOs) through CCD-based surveys, the number of known objects beyond Neptune has steadily increased. The LSST project is expected to increase the number of known TNOs by a factor of ten and to enable a large-scale statistical analysis of the outer architecture of the Solar System. As observed in the 2/3 mean-motion resonance between Pluto, the Plutinos, and Neptune, TNOs can be captured into resonant configurations. Neptune's outward migration is essential for understanding how such configurations formed. Using the REBOUND integrator, we investigate the capture of asteroids during Neptunes migration, assuming that its semi-major axis follows an exponential law. To calculate the probability of resonant capture of TNOs, Neptune migrates to its current location and encounters asteroids on eccentric orbits with random orientations. With future exploration of the TNO region, asteroids in mean-motion resonances and on highly inclined orbits may be observed. Therefore, we consider arbitrary orbital inclinations for the asteroids. For each 1° increment in orbital inclination, we integrate 1 000 initial conditions, determining the capture stages and the different final configurations. We identify capture-efficiency peaks for the resonances and conclude that retrograde TNOs have a higher probability of being captured in mean-motion resonance with Neptune.

This is a joint work with Helena Morais and Alexandre Correia.

Author: Gabriel Teixeira Guimaraes
SOKENDAI/NAOJ

Title: *"Dynamical instabilities in apsidally aligned systems"*

Abstract. Compact protoplanet systems are an outcome of oligarchic growth of planetesimals, with low-mass protoplanets with orbital separations of K about 10 mutual Hill radii. Those protoplanets evolve to Earth-mass bodies through giant impacts after gravitational instabilities are triggered. On the other hand, Kepler observations reveal older, non-resonant, and more massive systems with orbital separations clustered at K¿10 Hill radii, suggesting long-term stability despite similar compactness. In such compact systems, even small initial eccentricities quickly reduce the timescales required for a system to come to a close encounter, drastically reshaping its dynamical architecture, as a consequence of the onset of chaotic motion due to the overlap of Mean Motion Resonances (MMR). Nonetheless, results in the literature only studied the effect of initial eccentricities in systems with randomized longitudes of periapsis. In this work, we show how the longitudes of periapsis can affect the dynamical stability of eccentric, compact planetary systems. We show how systems in apsidal alignment can remain dynamically stable even up to larger eccentricities, leading planetary orbits to precess at the same rate as a rigid-body-like corotation, thus providing dynamical protection against. We also show how this apsidal corotation breaks down at larger eccentricities, leading to differential precession of orbits and consequently to dynamical instabilities.

Author: Giovanni Valsecchi
IAPS-INAF

Title: *"A numerical exploration of resonances in the lunar problem"*

Abstract. The Saros is an eclipse cycle of 223 synodic months; the study of the commensurabilities among the lunar months involved in this cycle allowed to find that the lunar orbit is very close to a set of 8 periodic orbits of the restricted circular 3-dimensional Sun-Earth-Moon problem, differing from each other for the phases, whose duration is 223 synodic months.

These periodic orbits are not the longest ones that can be found close to the lunar orbit; rather, the opposite is found numerically to be true.

This is not surprising in view of Poincaré's conjecture, according to which there should be infinitely many periodic orbits, of longer and longer duration, that should get closer and closer to the actual lunar motion.

It has already been found that, in the restricted circular, three-dimensional Sun-Earth-Moon three-body problem, the longer periodic orbits are arranged, in the eccentricity-inclination plane, in a characteristic pattern that is a deformation of the arrangement, in frequency space, of ratios of the durations of the lunar months.

In the light of the above considerations, we explore numerically the close vicinity of the lunar orbit in orbital elements space, in order to characterize the periodic orbits that can be found as close as possible to the real lunar orbit.

Author: Massimiliano Vasile
`Mechanical and Aerospace Engineering`
Title: *"Generative Astrodynamics: from the generation of periodic solutions to orbit determination in the cis-lunar environment"*

Abstract. The talk will introduce the concept of Generative Astrodynamics and present some applications to the generation of periodic orbits, the identification of stable and metastable equilibrium solutions and the determination of the orbit of objects in the cis-lunar environment. The talk will open with a brief explanation of what generative AI is, how it works and how it can be applied to the study of physical systems.

Author: Claudio Vidal Diaz
Universidad del Bío-Bío, Mathematics
Title: *"Periodic solutions of symmetric bi-Kepler perturbations and applications"*
Abstract. We investigate the existence of several families of symmetric periodic solutions as continuation of circular orbits of the bi-Kepler problem for certain symmetric differentiable perturbations using an appropriate set of Poincaré-Delaunay coordinates which are essential in our approach. We try separately two situations in an independent way, namely, when the unperturbed part corresponds to two decoupled Kepler problems in inertial cartesian coordinates and when it corresponds to two decoupled Kepler problems in rotating coordinates on R2. Finally, we apply these results to study the existence of periodic orbits of the planar three-body problem.